FROM THE YUKAWA TO THE EFIMOV ATTRACTION

Pascal Naidon,
RIKEN

INTERNATIONAL WORKSHOP ON QUANTUM MANY-BODY PROBLEMS IN PARTICLE, NUCLEAR, AND ATOMIC PHYSICS

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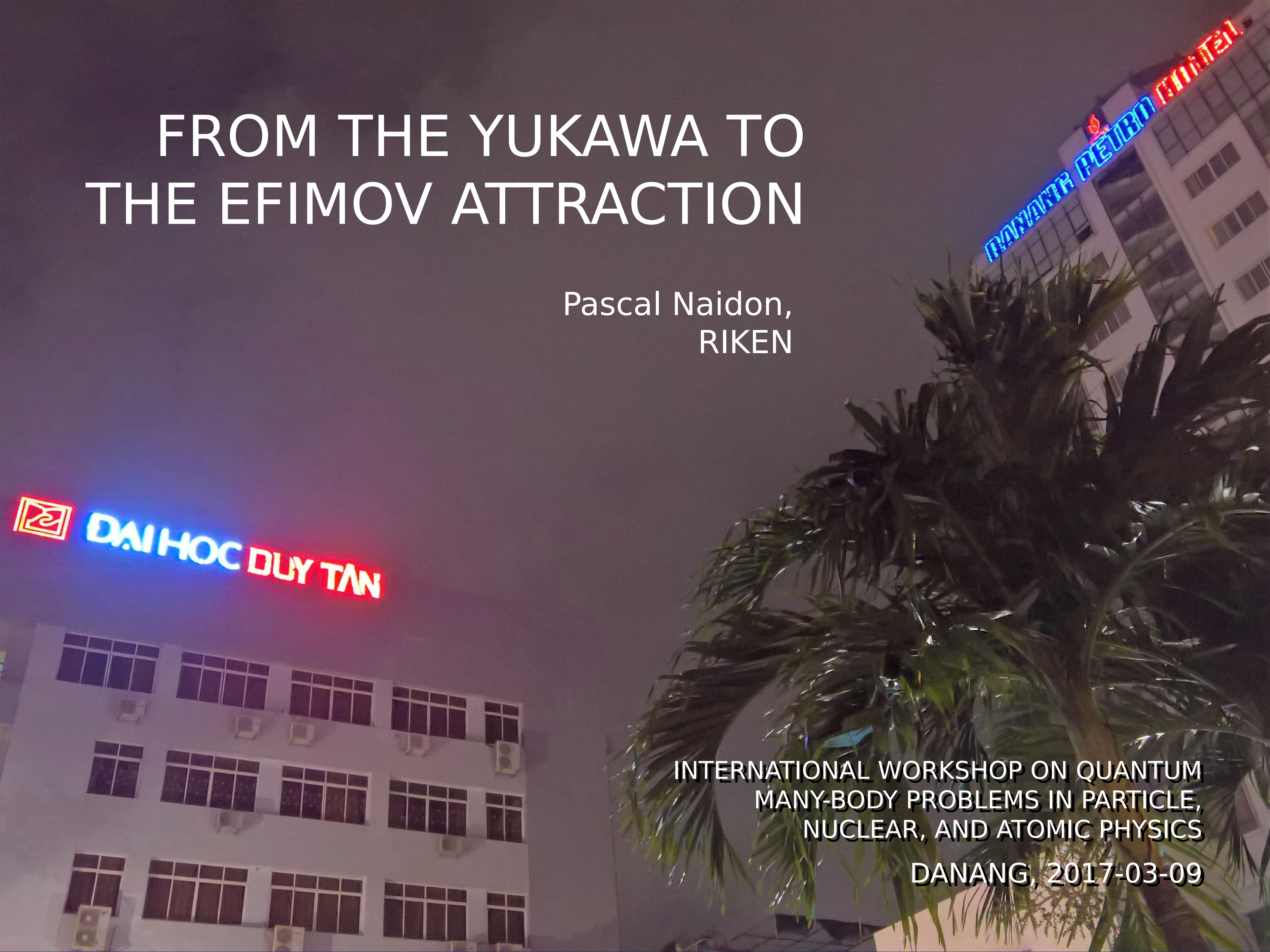
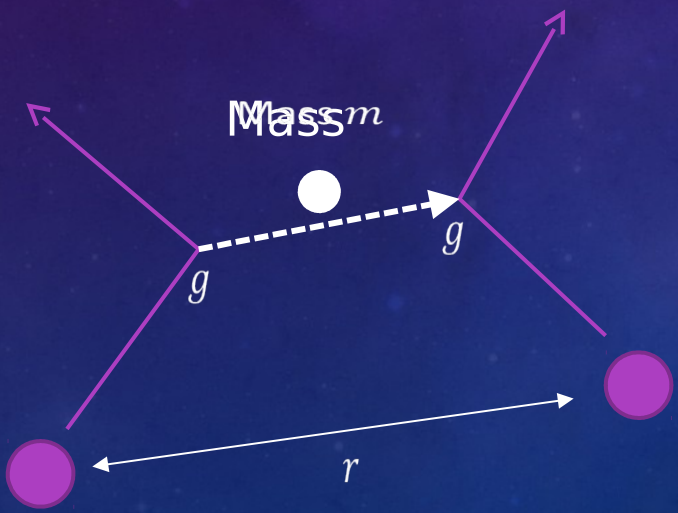


THE YUKAWA POTENTIAL

- In the 1930s, Hideki Yukawa showed that the exchange of a massive boson field between two particles induces a Coulomb potential exponentially screened by the boson mass.



Mediated potential

$$V(r) = -g^2 \frac{e^{-kmr}}{r}$$

THE YUKAWA POTENTIAL

- Coulomb potential of electromagnetism: exchange of photon ($m = 0$) between charged particles
- Tail of the nuclear force: exchange of mesons ($m \geq 135$ MeV) between nucleons

THE EFIMOV POTENTIAL

- In the 1970s, Vitaly Efimov showed that a universal three-body force arises between three resonantly-interacting particles.

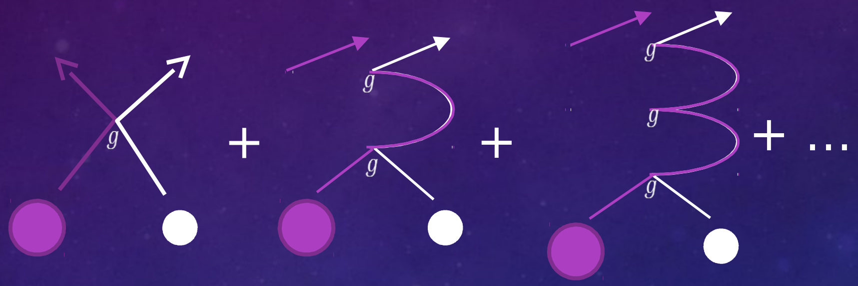
$$V(R) = -\frac{\hbar^2}{2M}\frac{1.006^2}{R^2}$$

$$V(r) = -\frac{\hbar^2}{2m}\frac{0.414^2}{r^2}$$

May also be viewed as a two-body mediated potential

RESONANT INTERACTION

Diagrammatic point of view

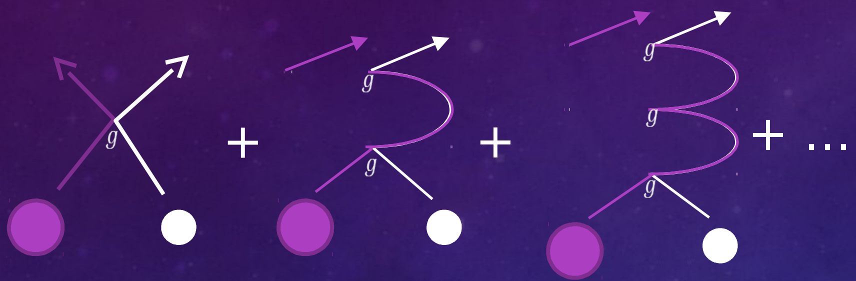


$$\underbrace{\frac{1}{g}}_{<0} + \underbrace{\sum_{\boldsymbol{k}}^{\Lambda} \frac{1}{\frac{\hbar^2 k^2}{2\mu}}}_{>0} = \frac{2\mu}{4\pi\hbar^2}\frac{1}{a}$$

Scattering length
Scattering length a
Reduced mass
Reduced mass $\mu = \left(\frac{1}{m} + \frac{1}{M}\right)^{-1}$

RESONANT INTERACTION

Diagrammatic point of view

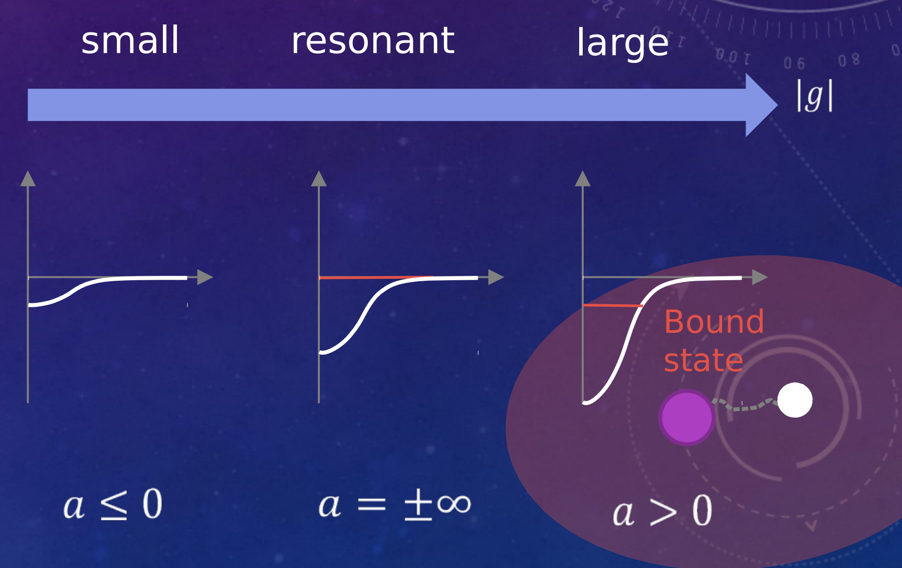


$$\underbrace{\frac{1}{g}}_{<0} + \underbrace{\sum_{\boldsymbol{k}}^{\Lambda} \frac{1}{\frac{\hbar^2 k^2}{2\mu}}}_{>0} = \frac{2\mu}{4\pi\hbar^2}\frac{1}{a}$$

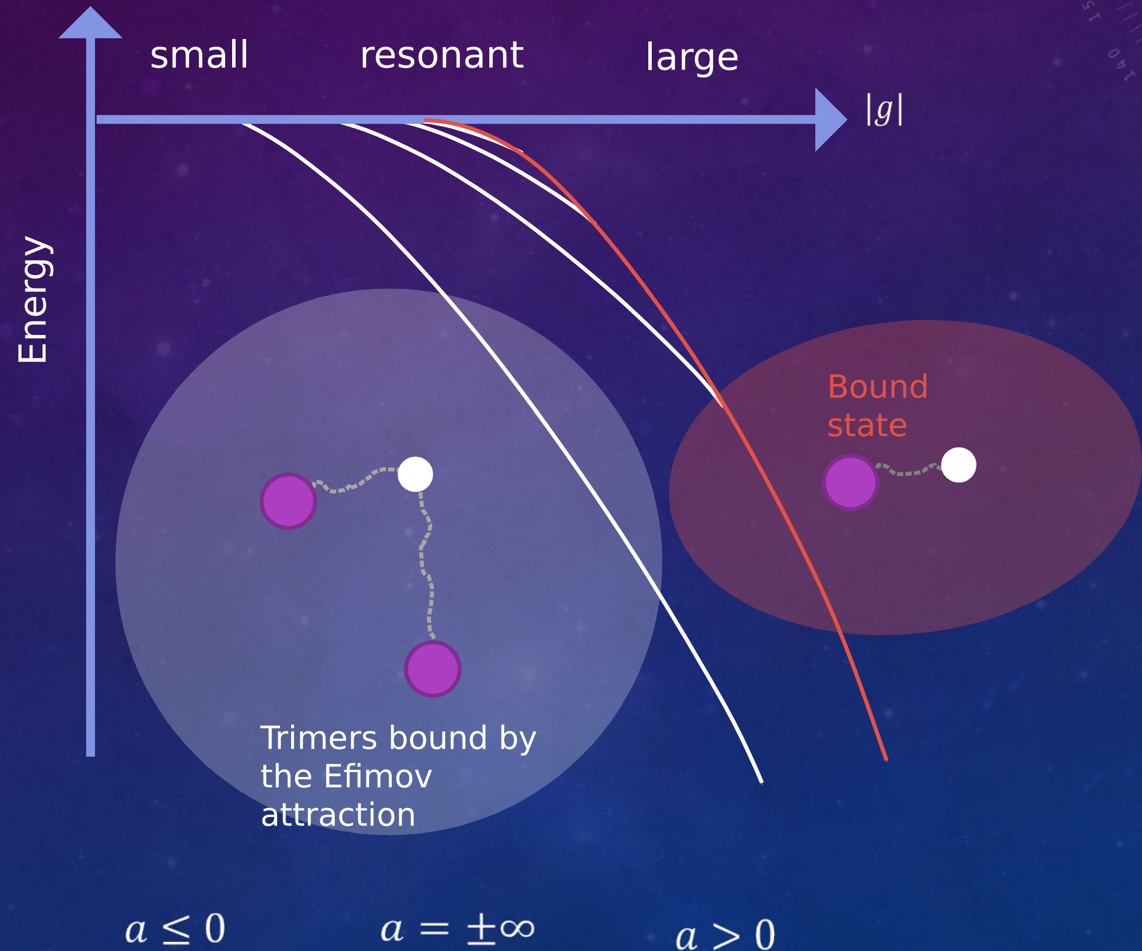
Scattering length

Reduced mass $\mu = \left(\frac{1}{m} + \frac{1}{M}\right)^{-1}$

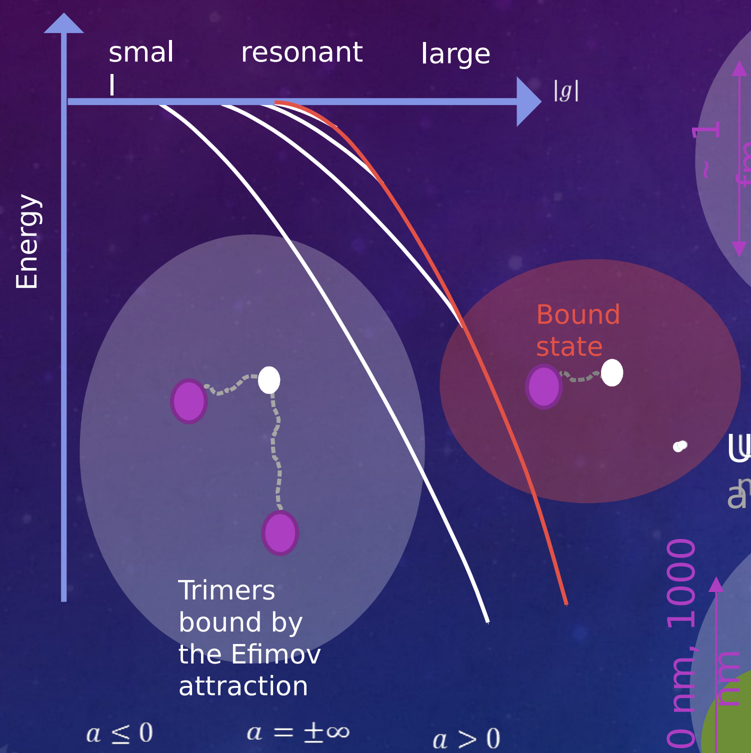
"Potential" point of view



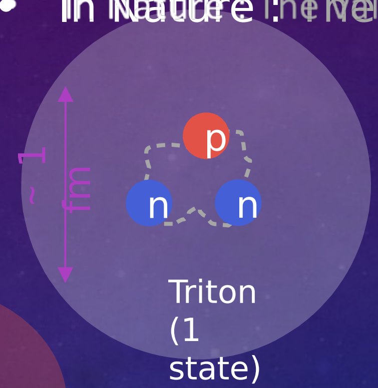
RESONANT INTERACTION

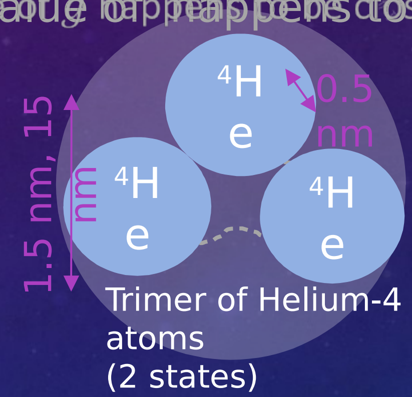


RESONANT INTERACTION



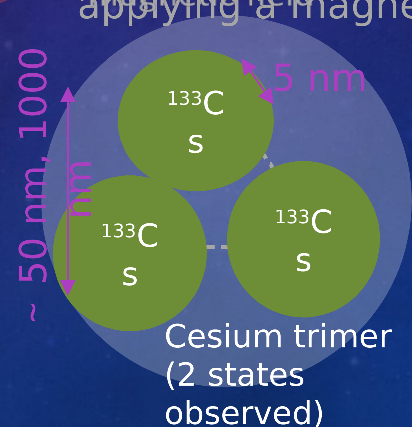
- In Nature : The value of happens to be close to reson

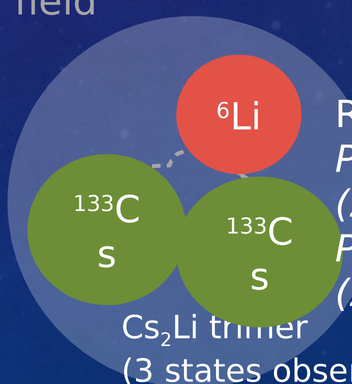




Recently observed *Science, 348, 551 (2015)*

- Ultra-cold atoms : The value of can be adjusted by applying a magnetic field





Recently observed
PRL 112, 250404 (2014)
PRL 113, 240402 (2014)

SYSTEMS EXHIBITING THE EFIMOV ATTRACTION

arXiv:1610.09805

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Review

Efimov physics: a review

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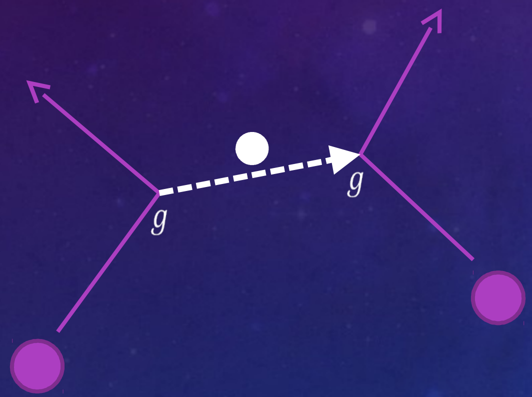
Abstract

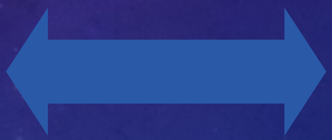
This article reviews theoretical and experimental advances in Efimov physics, an array of quantum few-body and many-body phenomena arising for particles interacting via short-range resonant interactions, that is based on the appearance of a scale-invariant three-body attraction theoretically discovered by Vitaly Efimov in 1970. This three-body effect was originally proposed to explain the binding of nuclei such as the triton and the Hoyle state of carbon-12, and later considered as a simple explanation for the existence of some halo nuclei. It was subsequently evidenced in trapped ultra-cold atomic clouds and in diffracted molecular beams of gaseous helium. These experiments revealed that the previously undetermined three-body parameter introduced in the Efimov theory to stabilise the three-body attraction typically scales with the range of atomic interactions. The few- and many-body consequences of the Efimov attraction have been since investigated theoretically, and are expected to be observed in a broader spectrum of physical systems.

MOTIVATION

Connection between Yukawa and Efimov mediated potentials?

Yukawa



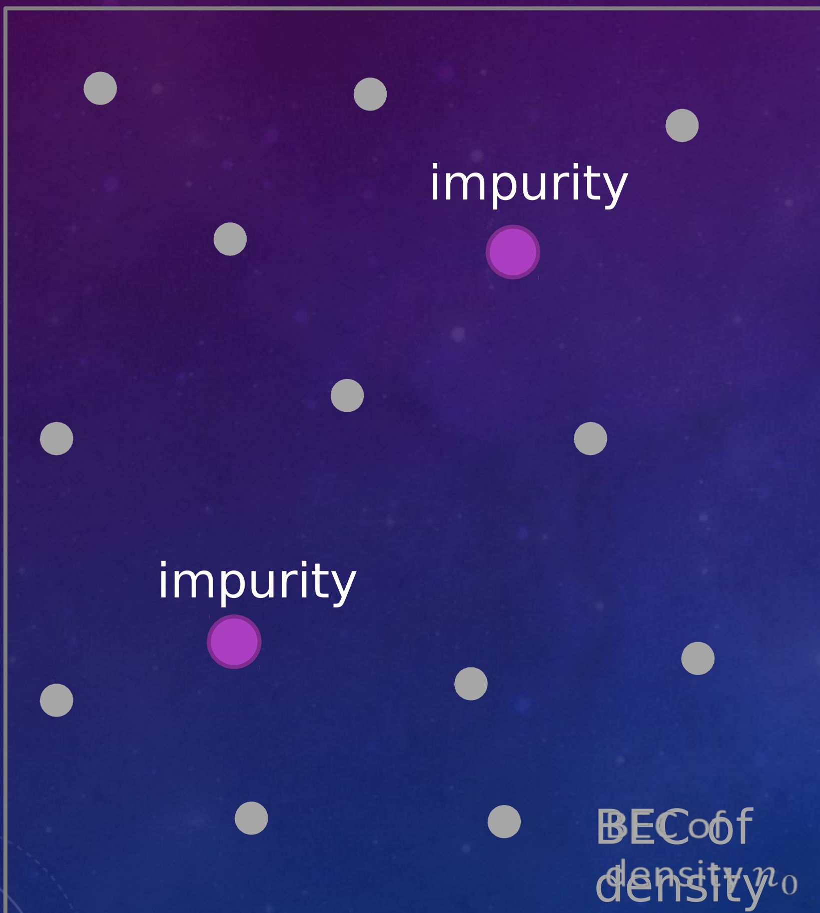


Efimov

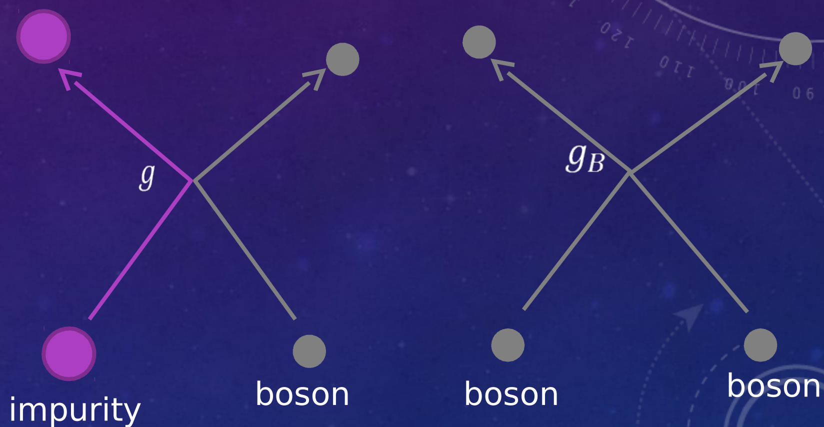
Many-body
Bosons are
created/absorbed

Three-body
Boson always
there

IMPURITIES IN A BOSE-EINSTEIN CONDENSATE



Interactions



$g < 0$ can be large

(attraction)

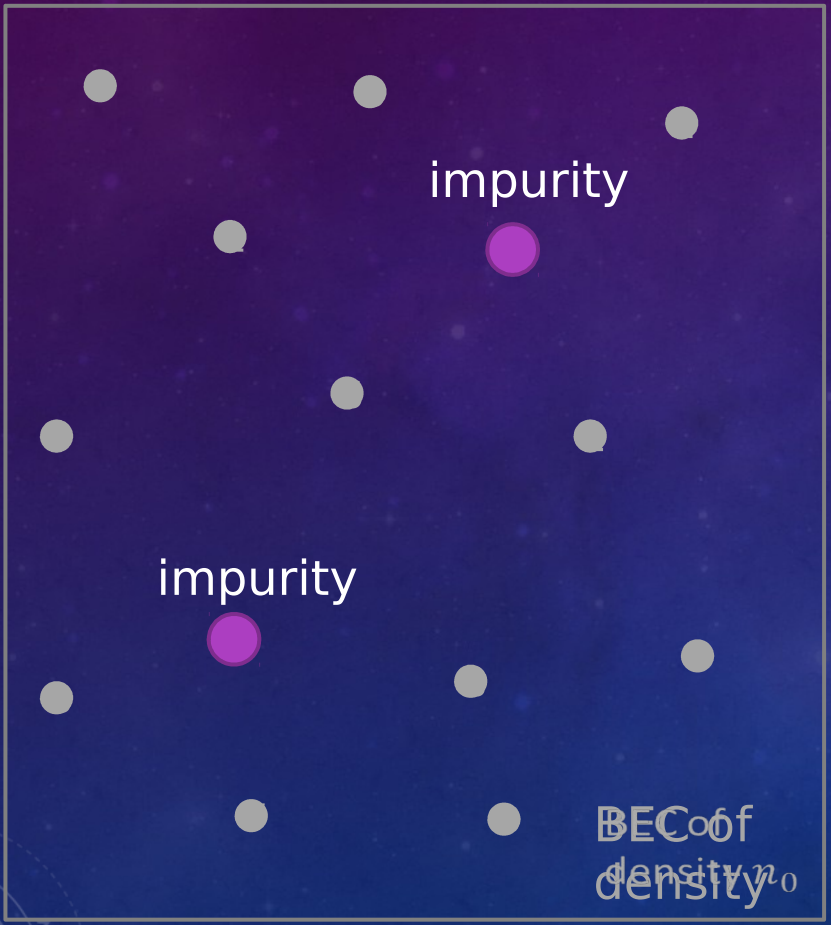
$g_B > 0$ is small

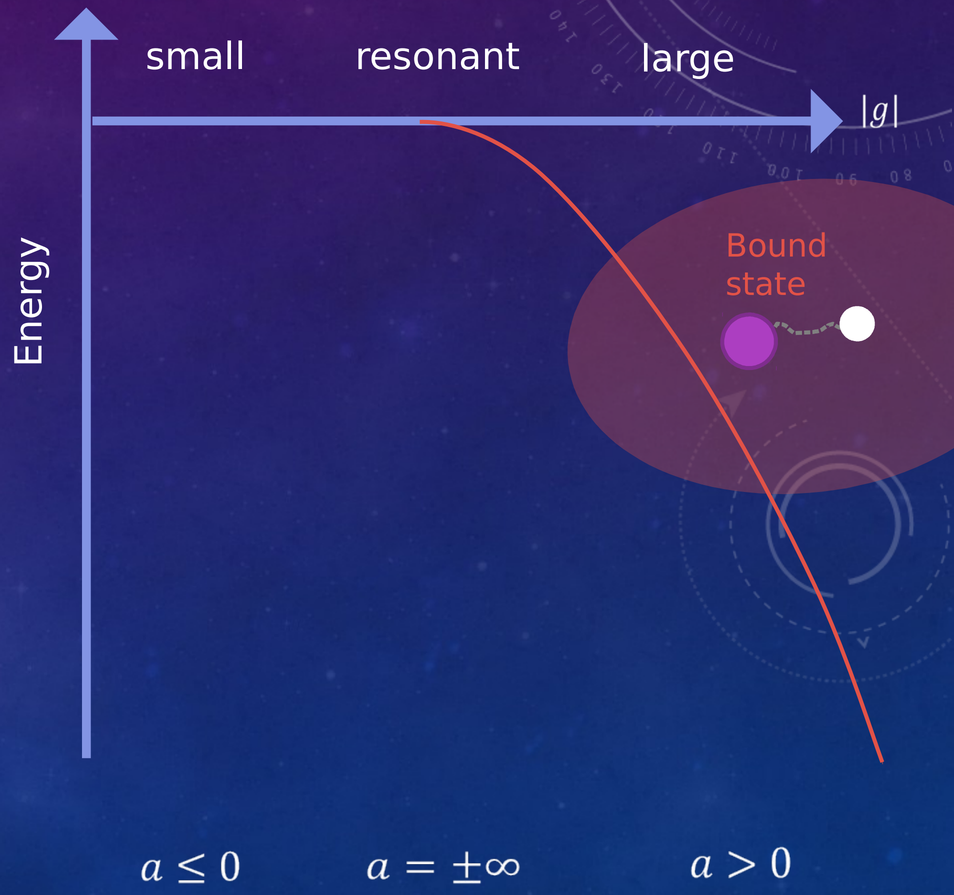
(weak repulsion)

$\boxed{n_0 a_B^3 \ll 1}$

Neglect direct interactions between impurities

IMPURITIES IN A BOSE-EINSTEIN CONDENSATE





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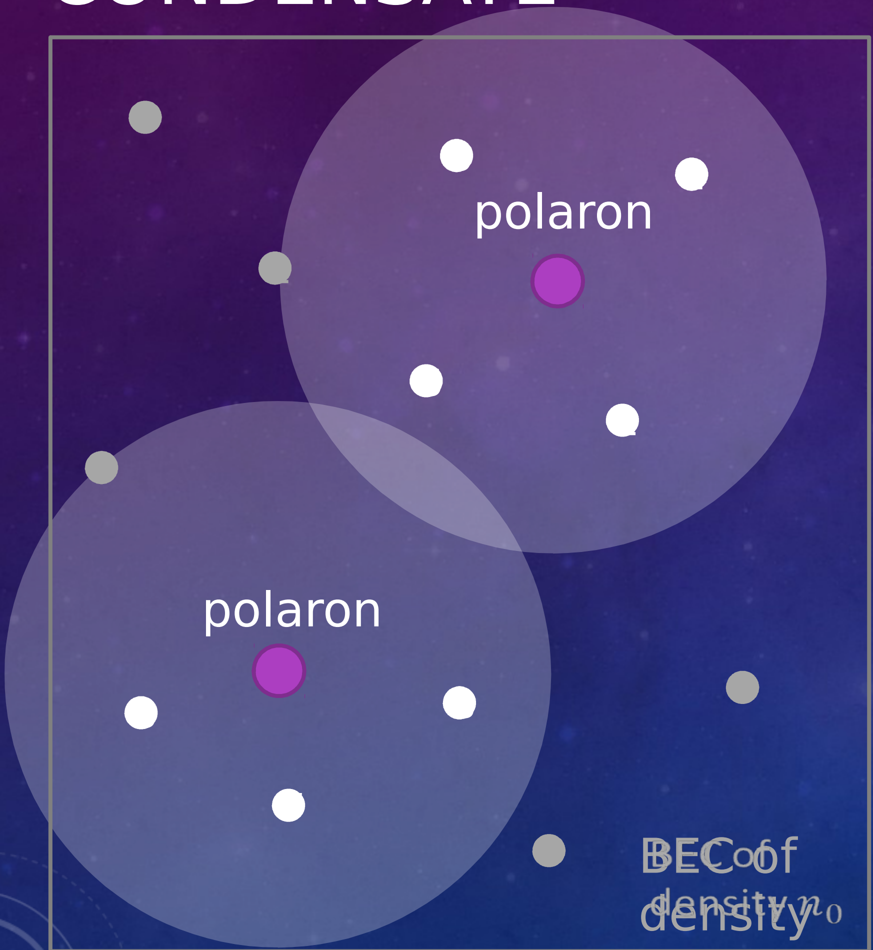

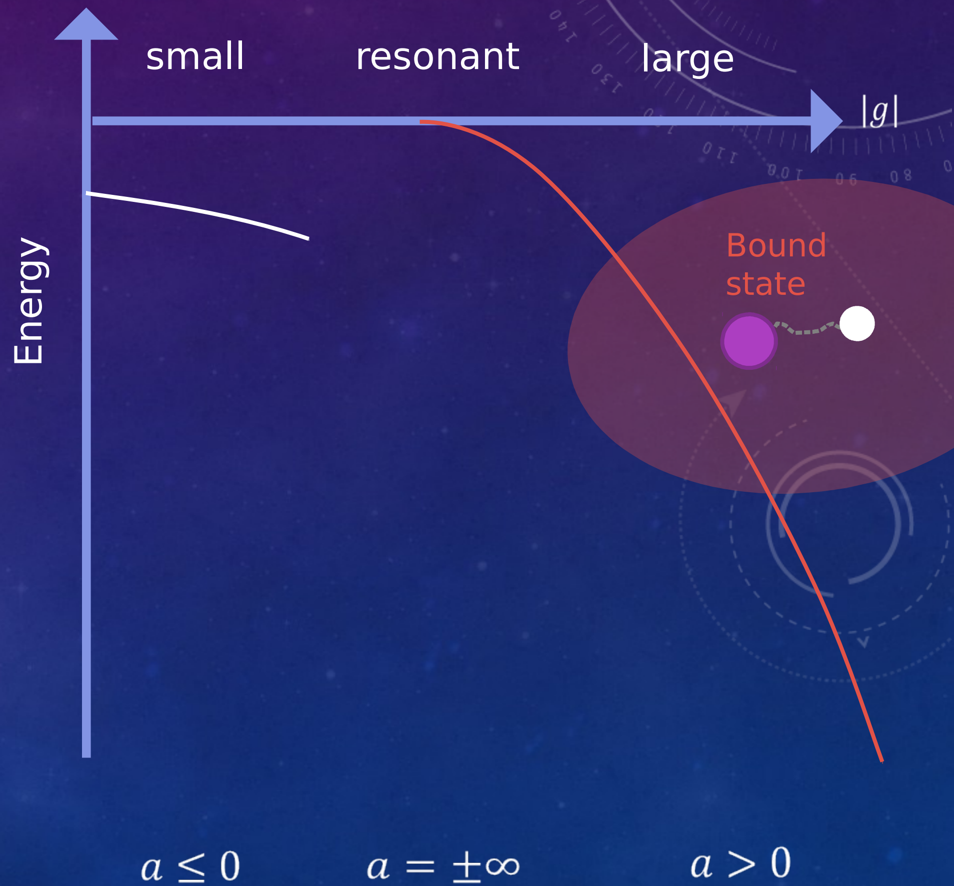

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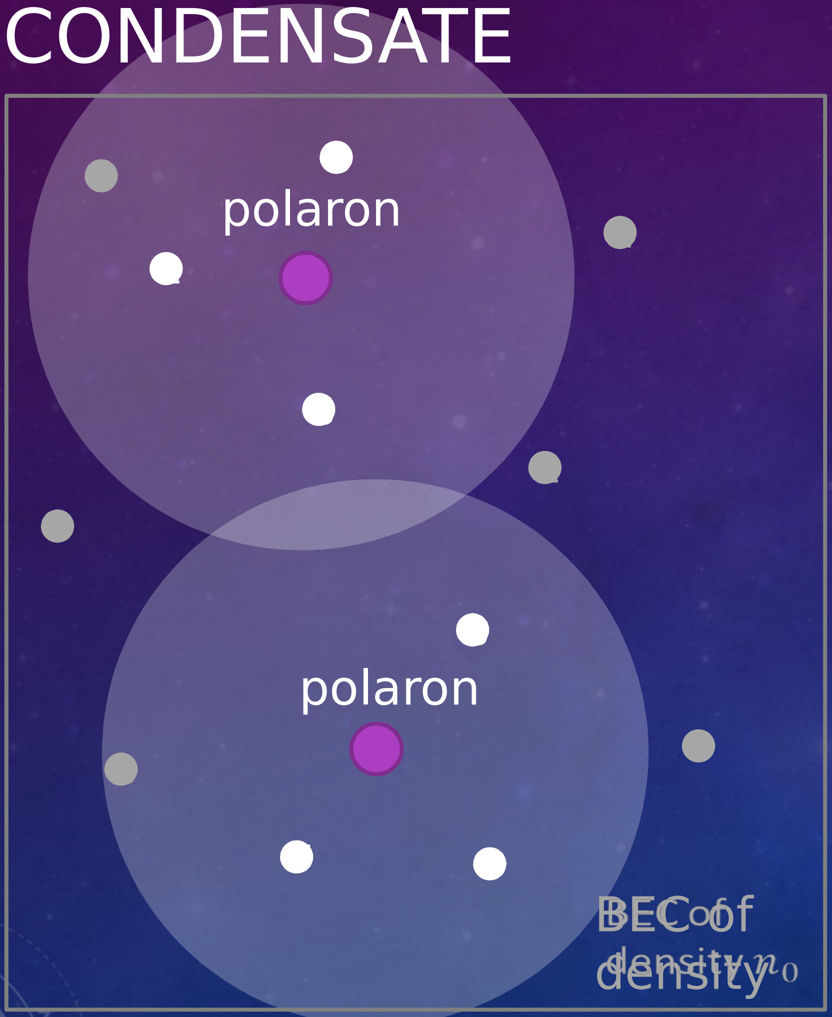

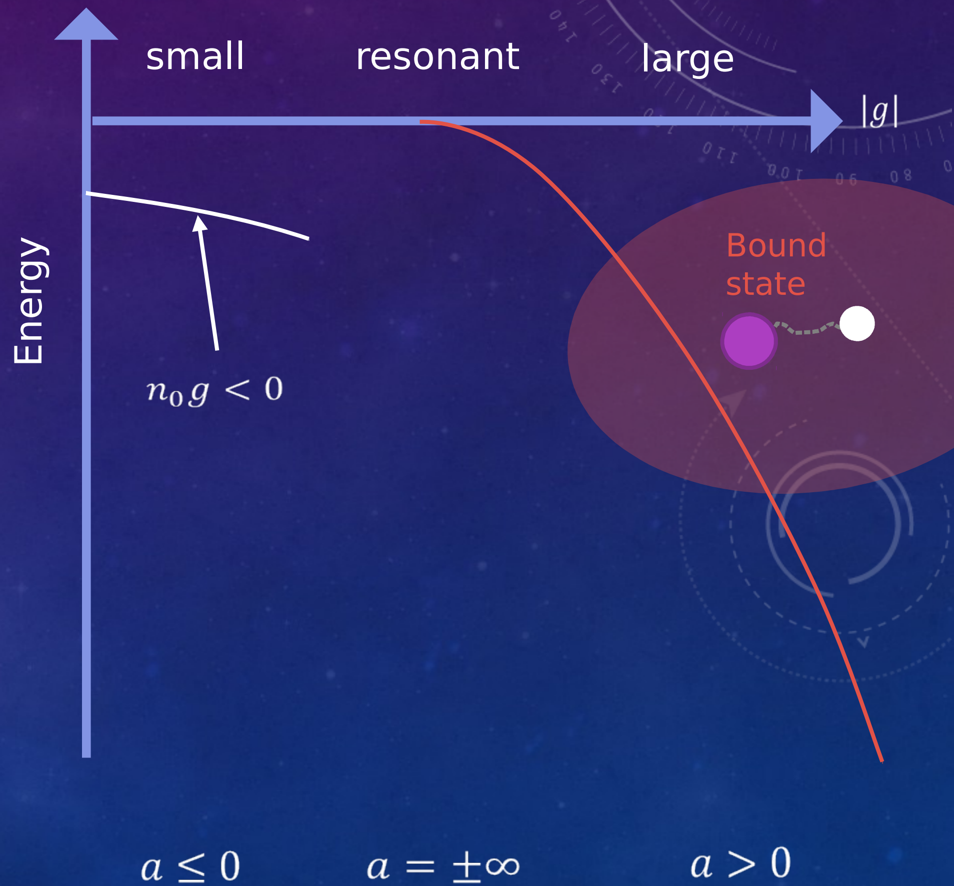

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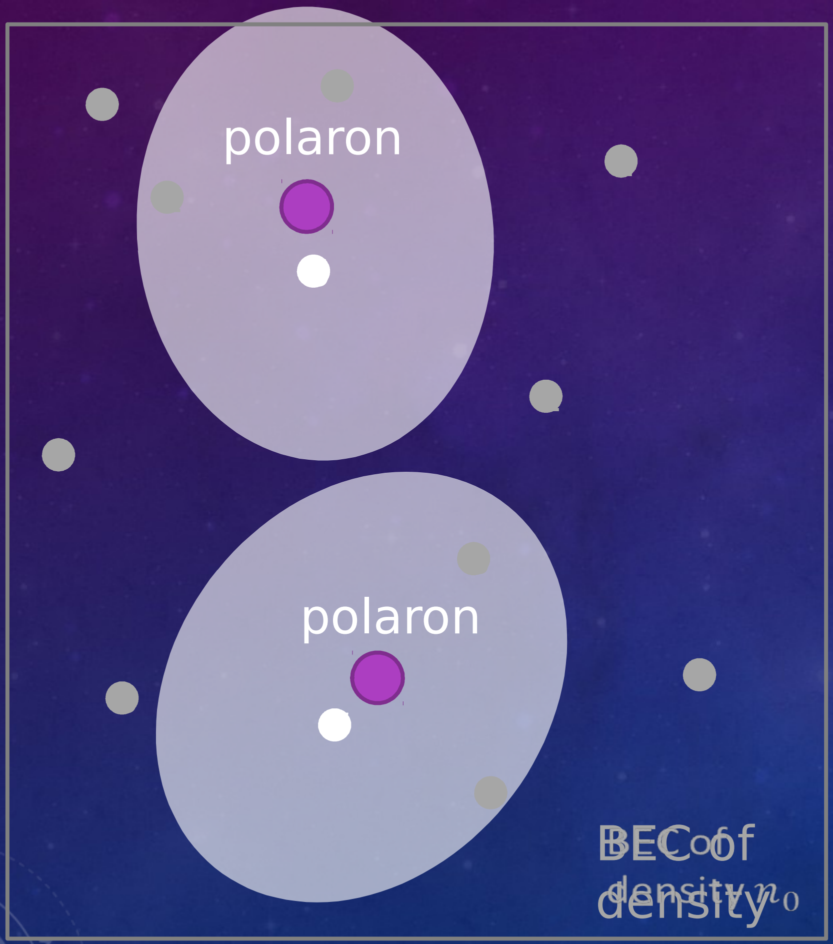
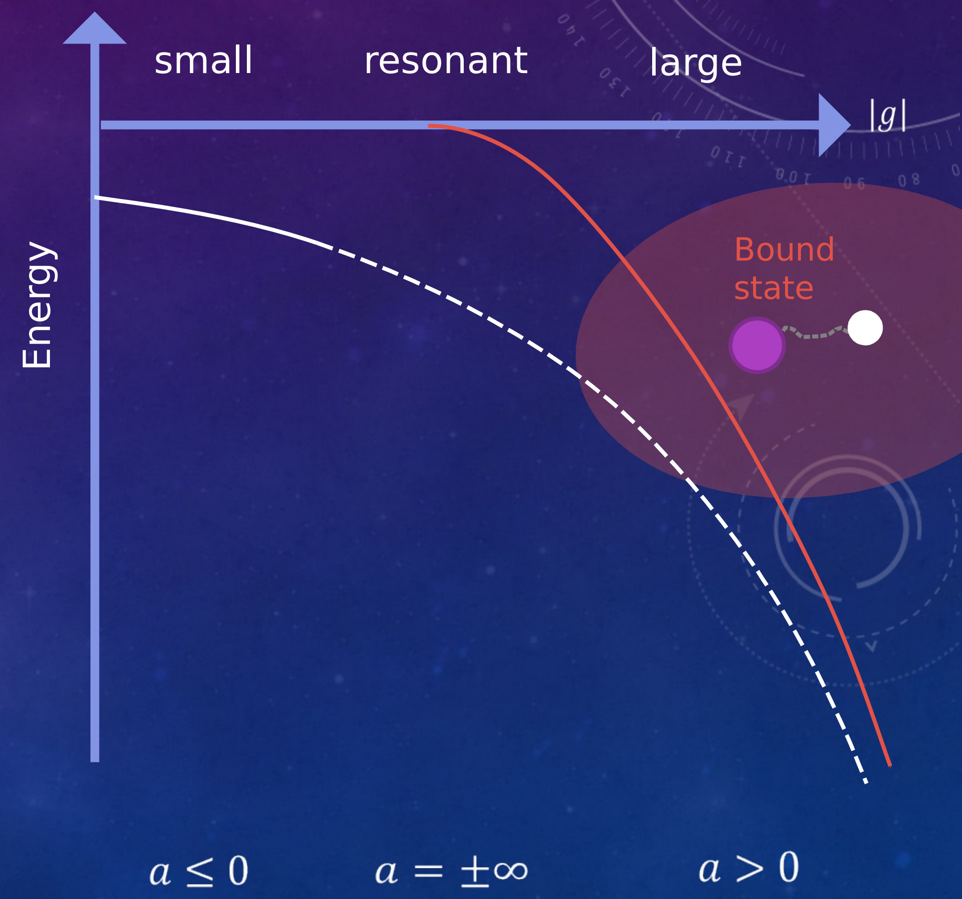
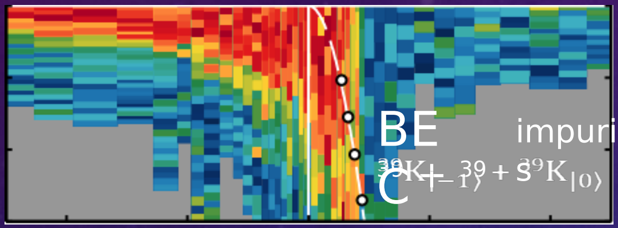


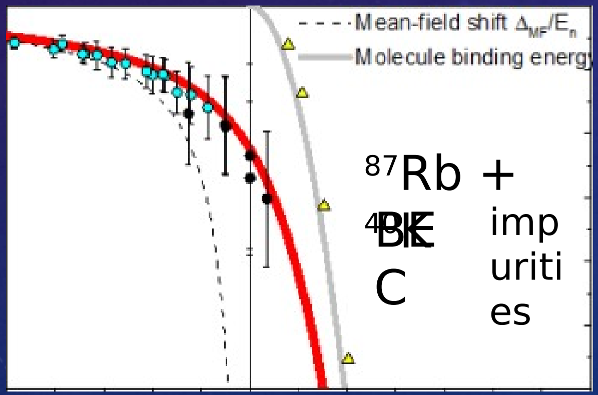


IMPURITIES IN A BOSE-EINSTEIN CONDENSATE

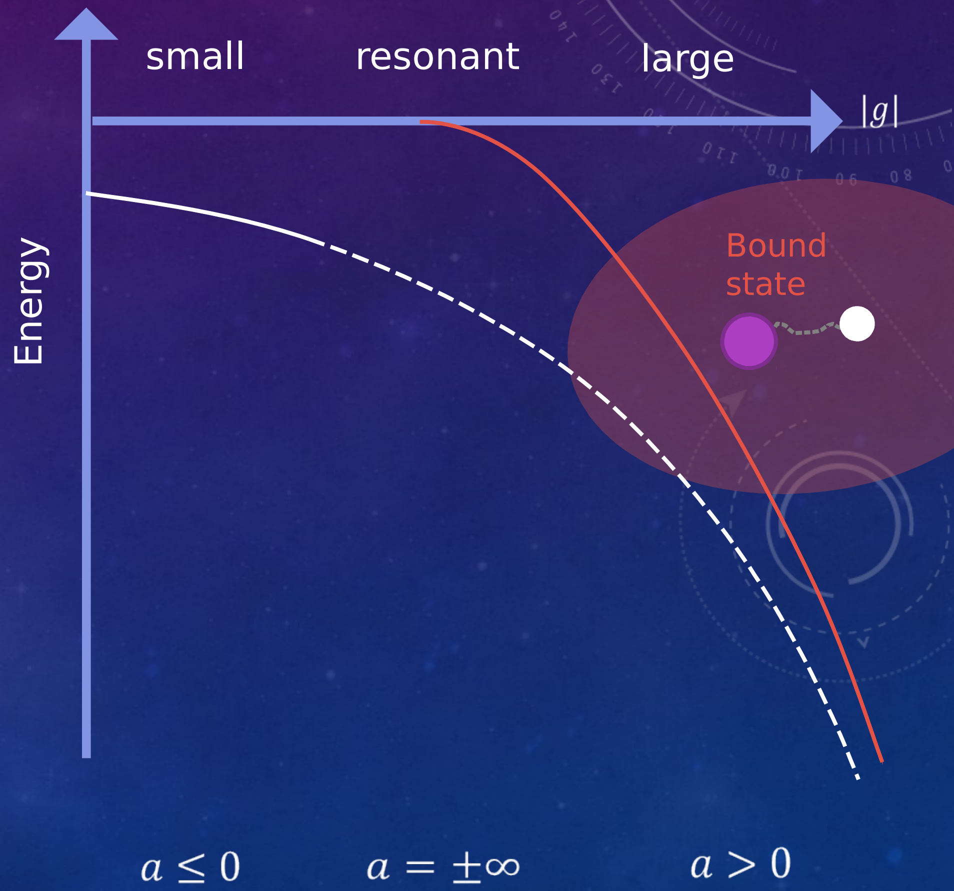
Bose polaron recently observed



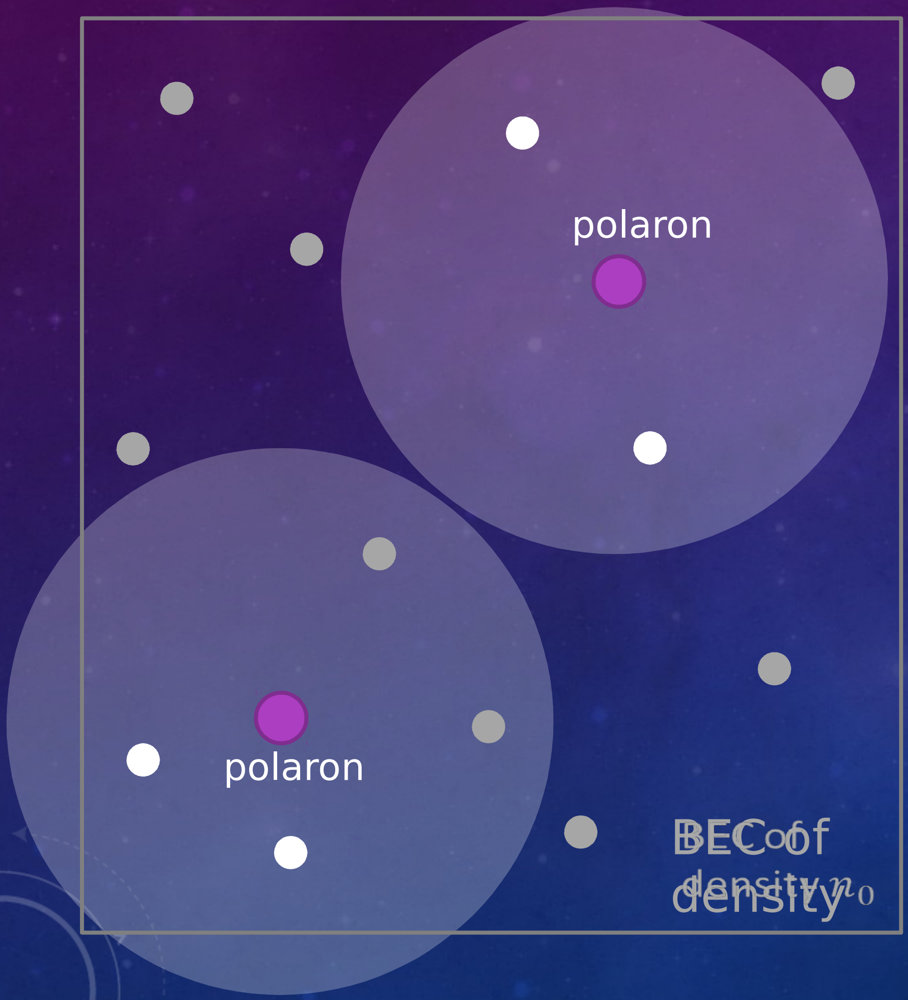
Jørgensen et al, PRL 117, 055302 (2016)



Ming-Guang Hu et al, PRL 117, 055301 (2016)



POLARONIC INTERACTION



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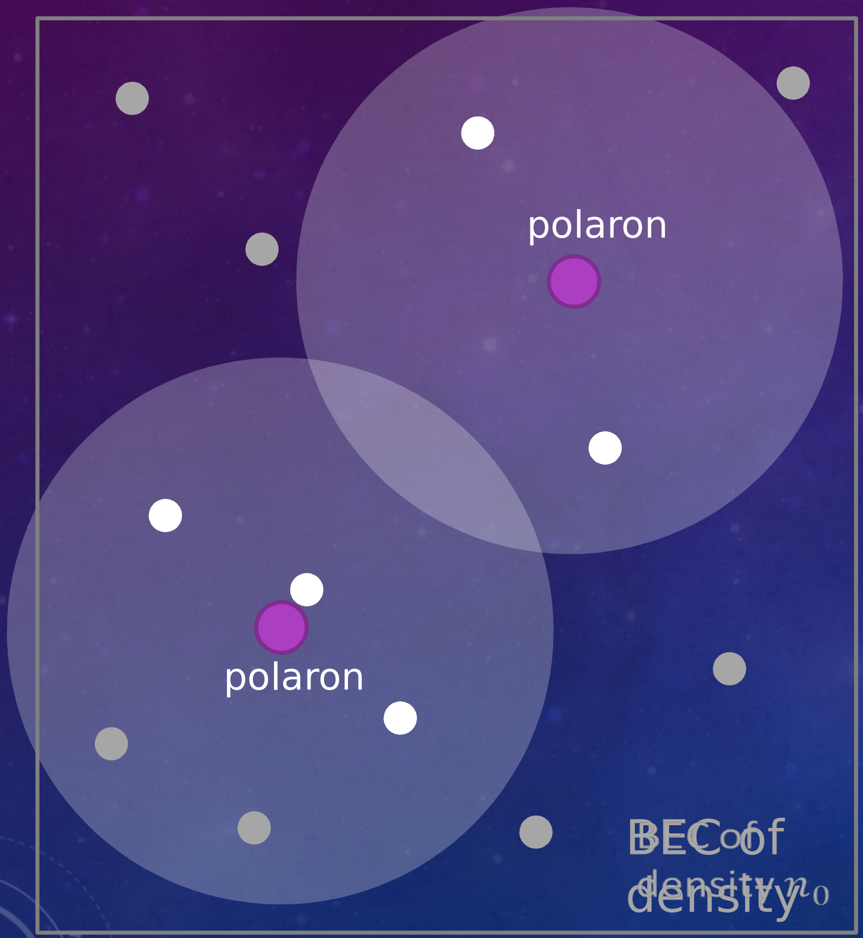

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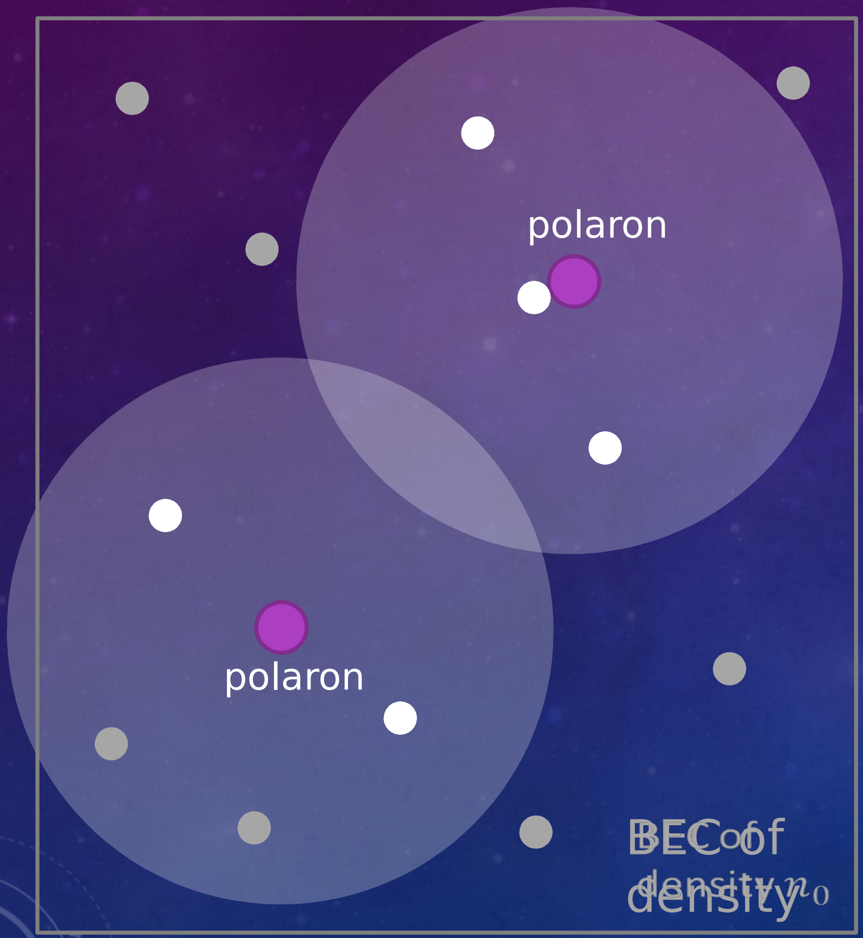

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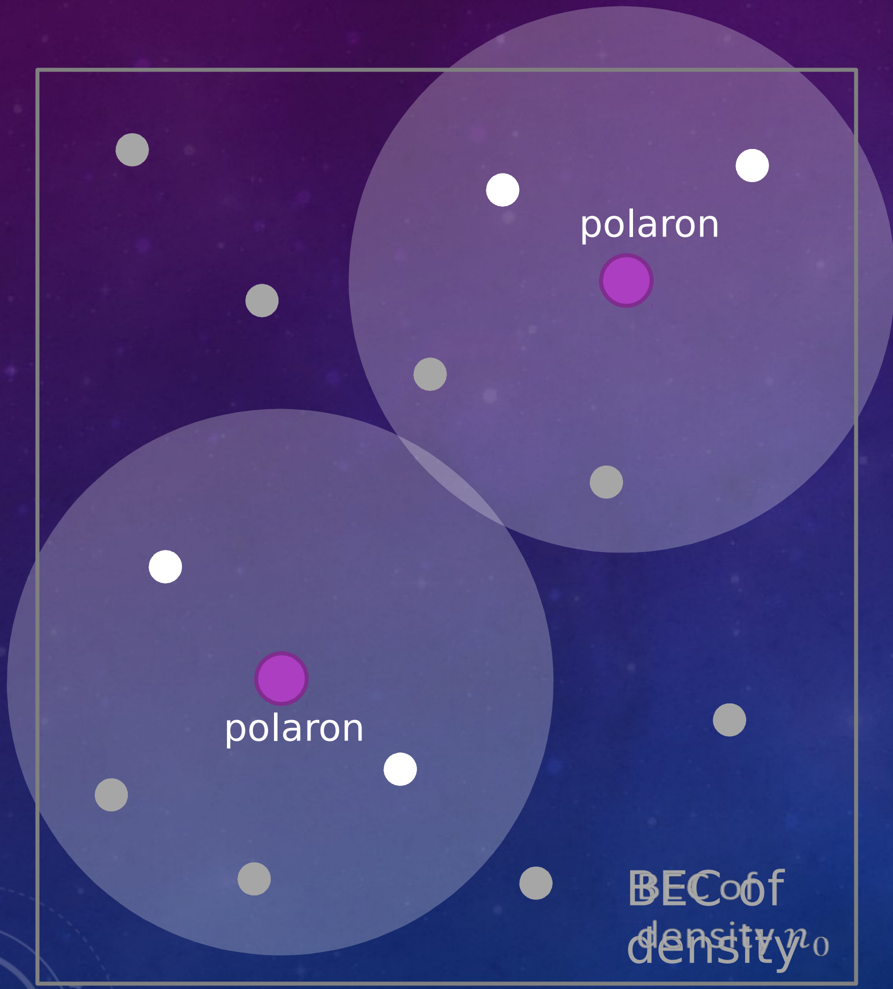

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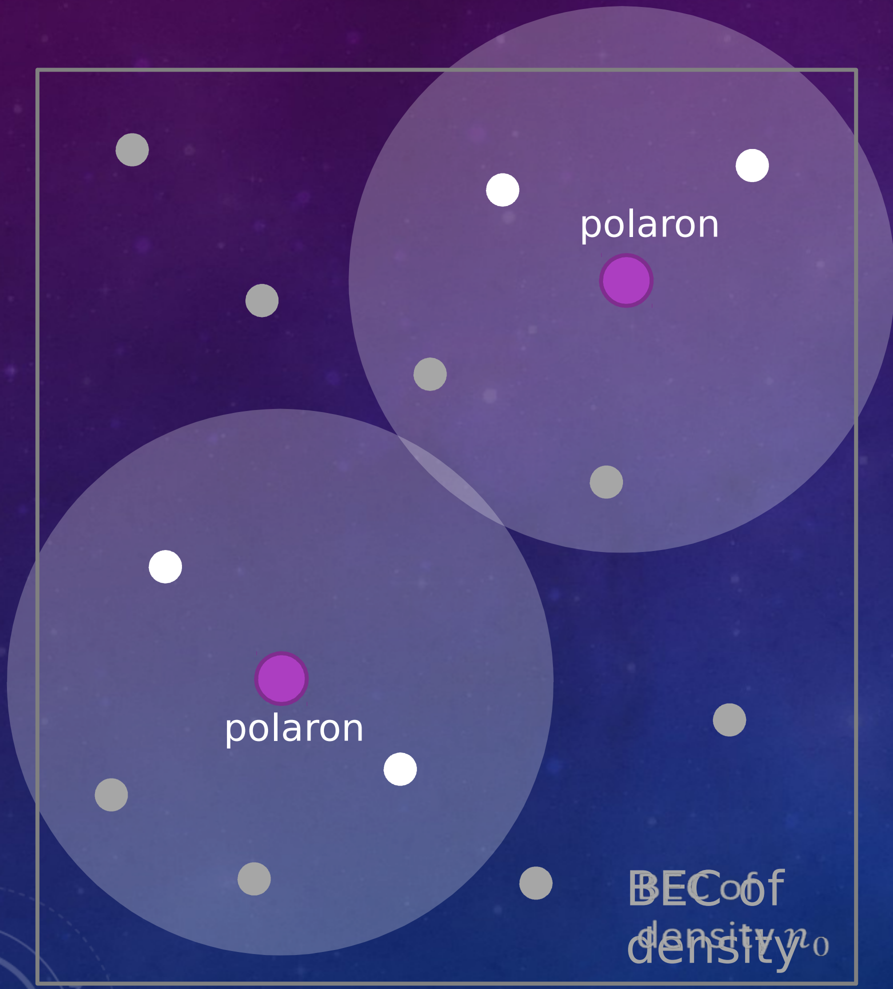
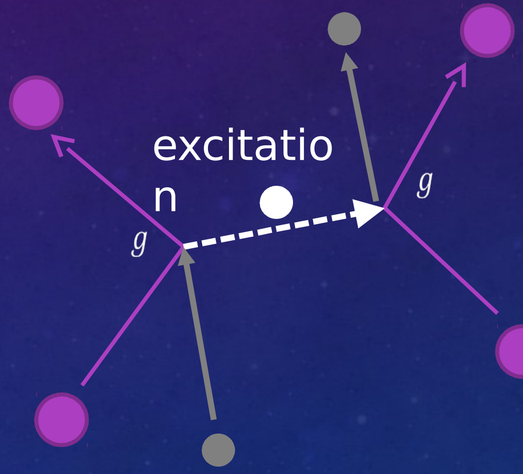


The (Bogoliubov) quasi-particles excitations of the BEC can mediate a Yukawa interaction



To second-order in perturbation theory:

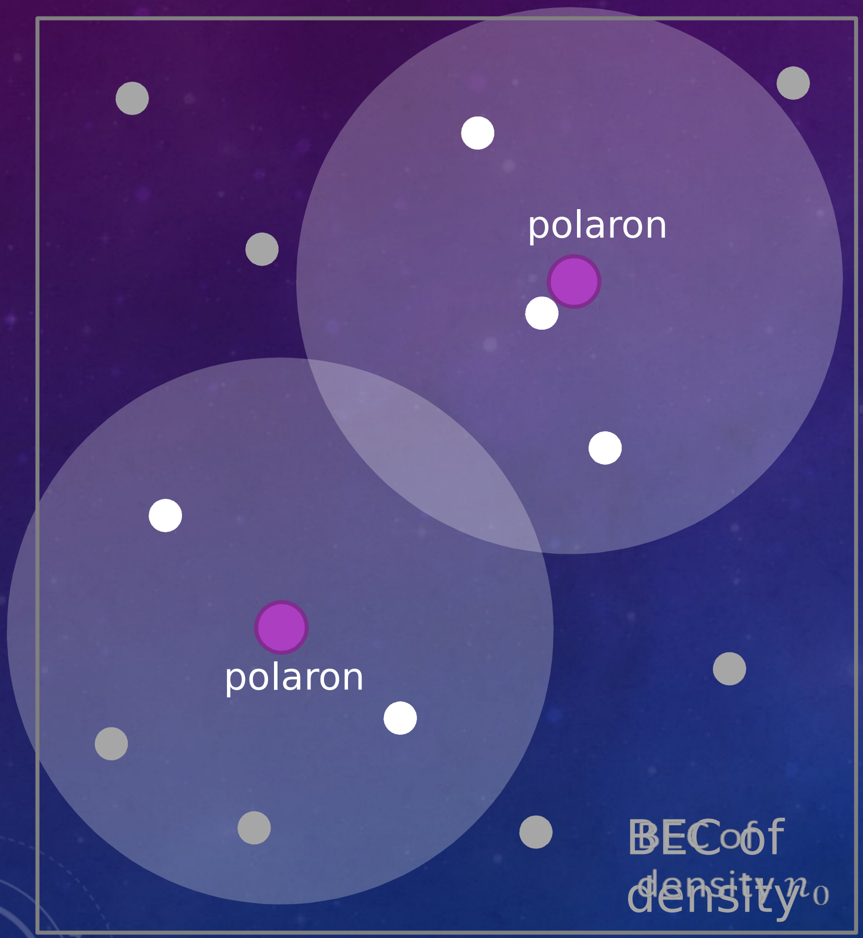
$$V(r) \propto -g^2 n_0 \frac{e^{-\sqrt{2}r/\xi}}{r}$$

BEC coherence length

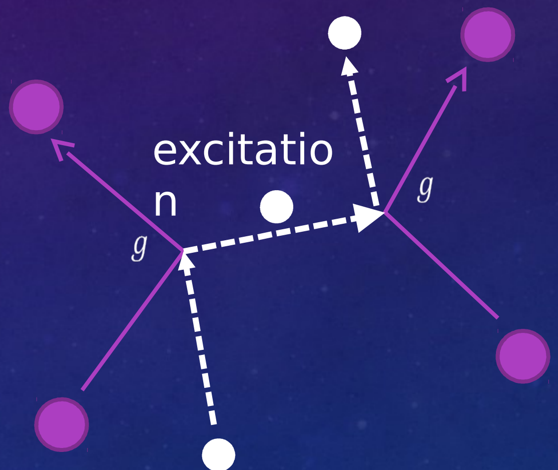
$$\xi = \frac{1}{\sqrt{8\pi n_0 a_B}}$$

Ex: Helium-3 impurities in Helium-4
J. Bardeen, G. Baym, and D. Pines, Phys Rev 156, 207 (1967)

POLARONIC INTERACTION



The (Bogoliubov) quasi-particles excitations of the BEC can also mediate an Efimov interaction



Non-perturbative

NON-PERTURBATIVE METHOD: TRUNCATED BASIS

Bosons

Impurities

Impurity-boson

$$H = \sum_k \epsilon_k b_k^\dagger b_k + \frac{g_B}{2V}\sum_{k,k',p} b_{k'-p}^\dagger b_{k+p}^\dagger b_k b_{k'} + \sum_k \varepsilon_k c_k^\dagger c_k + \frac{g}{V}\sum_{k,k',p} b_{k'-p}^\dagger c_{k+p}^\dagger c_k b_{k'}$$

$$\epsilon_k = \frac{\hbar^2 k^2}{2m}$$

$$\varepsilon_k = \frac{\hbar^2 k^2}{2M}$$

Bogoliubov appproximation

$b_0 = \sqrt{N_0}$ condensate

$b_k = u_k \beta_k - v_k \beta_k^\dagger$ Bogoliubov excitation

$$H = E_0 + \sum_k E_k \beta_k^\dagger \beta_k + \sum_k (\varepsilon_k + g n_0) c_k^\dagger c_k + \sqrt{N_0}\frac{g}{V}\sum_{k,p}(u_p\beta_{-p}^\dagger - v_p\beta_p)c_{k+p}^\dagger c_k + h.c.$$

Fröhlich

$$+\frac{g}{V}\sum_{k,k',p}(u_{k'-p}u_{k'}\beta_{k'-p}^\dagger\beta_{k'} + v_{k'-p}v_{k'}\beta_{p-k'}\beta_{-k'}^\dagger)c_{k+p}^\dagger c_k$$

$$+\frac{g}{V}\sum_{k,k',p}(u_{k'-p}v_{k'}\beta_{k'-p}^\dagger\beta_{-k'}^\dagger + v_{k'-p}u_{k'}\beta_{p-k'}\beta_{k'})c_{k+p}^\dagger c_k$$

Scattering

Bogoliubov excitation energy

$$E_k = \sqrt{\epsilon_k(\epsilon_k + 2g_B n_0)}$$

NON-PERTURBATIVE METHOD: TRUNCATED BASIS

$$|\Psi\rangle = \left(\sum_q \alpha_q c_q^\dagger c_{-q}^\dagger + \sum_{q,q'} \alpha_{q,q'} c_q^\dagger c_{q'}^\dagger \beta_{-q-q'}^\dagger + \sum_{q,q',q''} \alpha_{q,q',q''} c_q^\dagger c_{q'}^\dagger \beta_{q''}^\dagger \beta_{-q-q'-q''}^\dagger + \cdots\right)|\Phi\rangle$$

Impurity creation operator

Excitation creation operator

BEC ground state

EQUATIONS

Coupled equations for F_q and $\alpha_{q,q'}$

Zero-range limit (cutoff $\Lambda \to \infty$)

$$F_q = g\frac{1}{V}\sum_k u_k \alpha_{q,k-q}$$

Three-body Integral equation

$$\frac{1}{T_q(E)}F_q + \frac{1}{V}\sum_k \frac{u_k^2 F_{k-q}}{E_k + \varepsilon_{|k-q|} + \varepsilon_q - E} = \frac{2n_0}{2\varepsilon_q - E}F_q$$

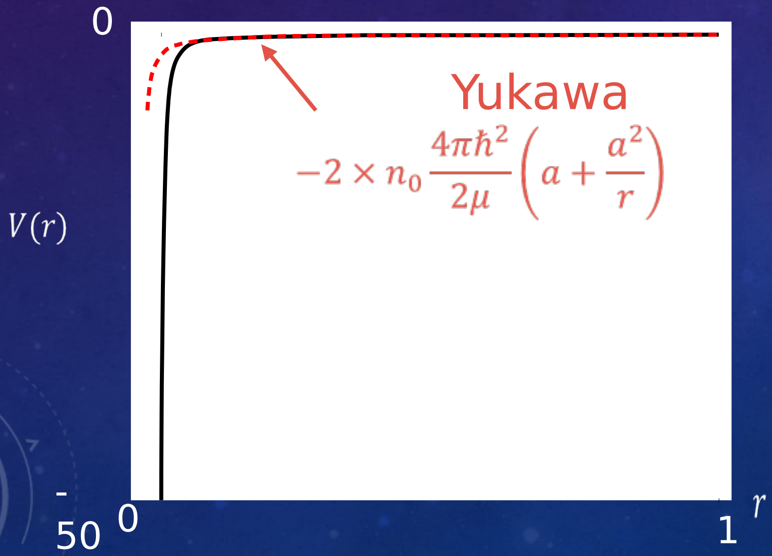
$$\frac{1}{T_q(E)} = \frac{2\mu}{4\pi\hbar^2}\frac{1}{a} + \frac{1}{V}\sum_k\left(\frac{u_k^2}{E_k + \varepsilon_{|k-q|} + \varepsilon_q - E} - \frac{1}{\epsilon_k + \varepsilon_k}\right)$$

RESULT: POLARONIC POTENTIAL

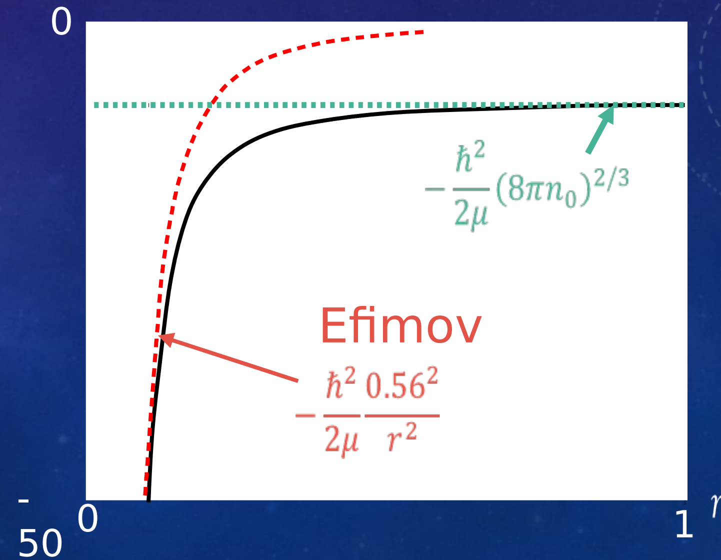
Effective potential (Born-Oppenheimer) between polarons:

$$V(r) = \frac{\hbar^2 \kappa^2}{2\mu} \qquad \frac{1}{a} - \kappa + \frac{1}{r} e^{-\kappa r} + \frac{8\pi n_0}{\kappa^2} = 0 \qquad (a_B \to 0)$$

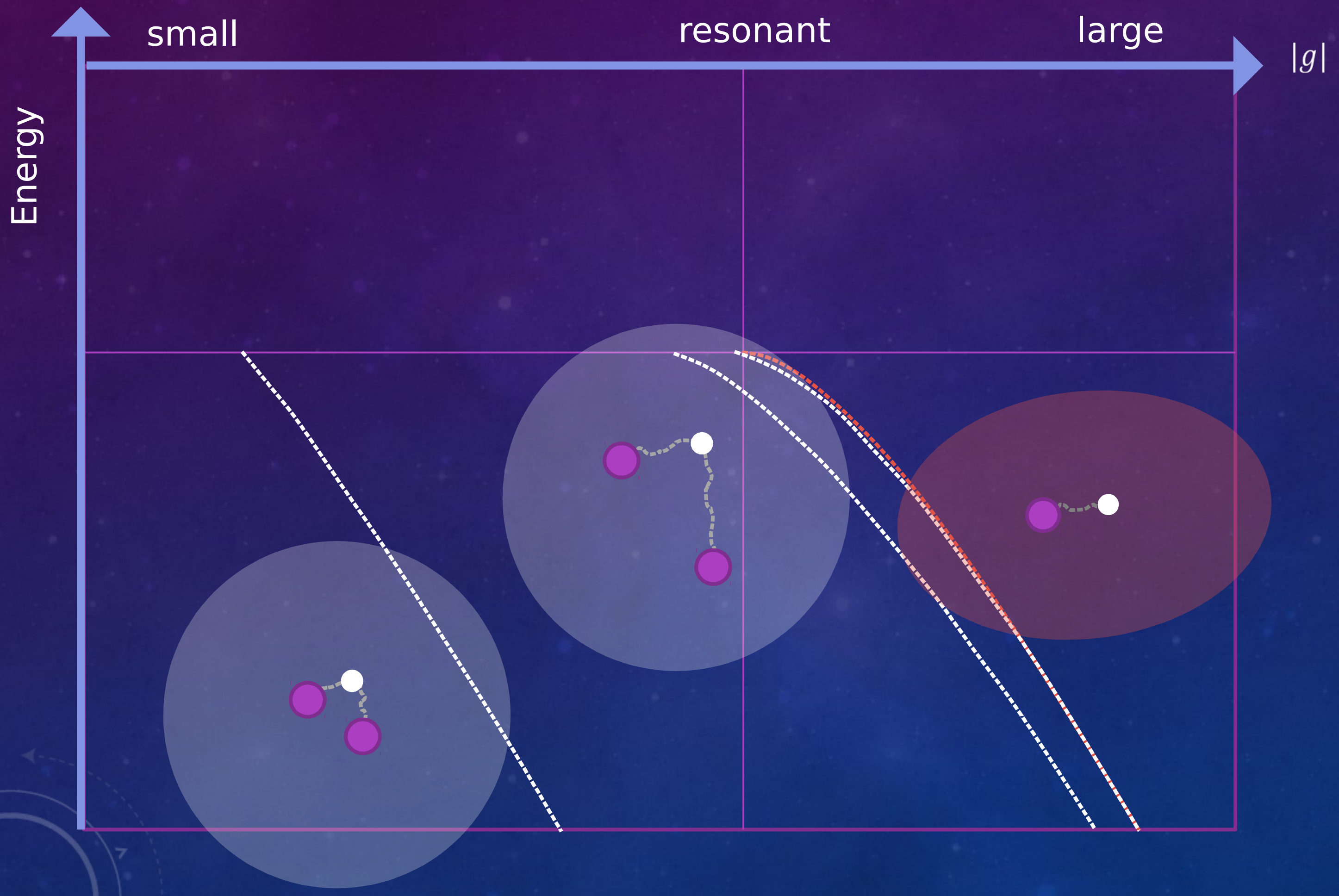
For small $a \lesssim 0$



At resonance $a = \pm\infty$



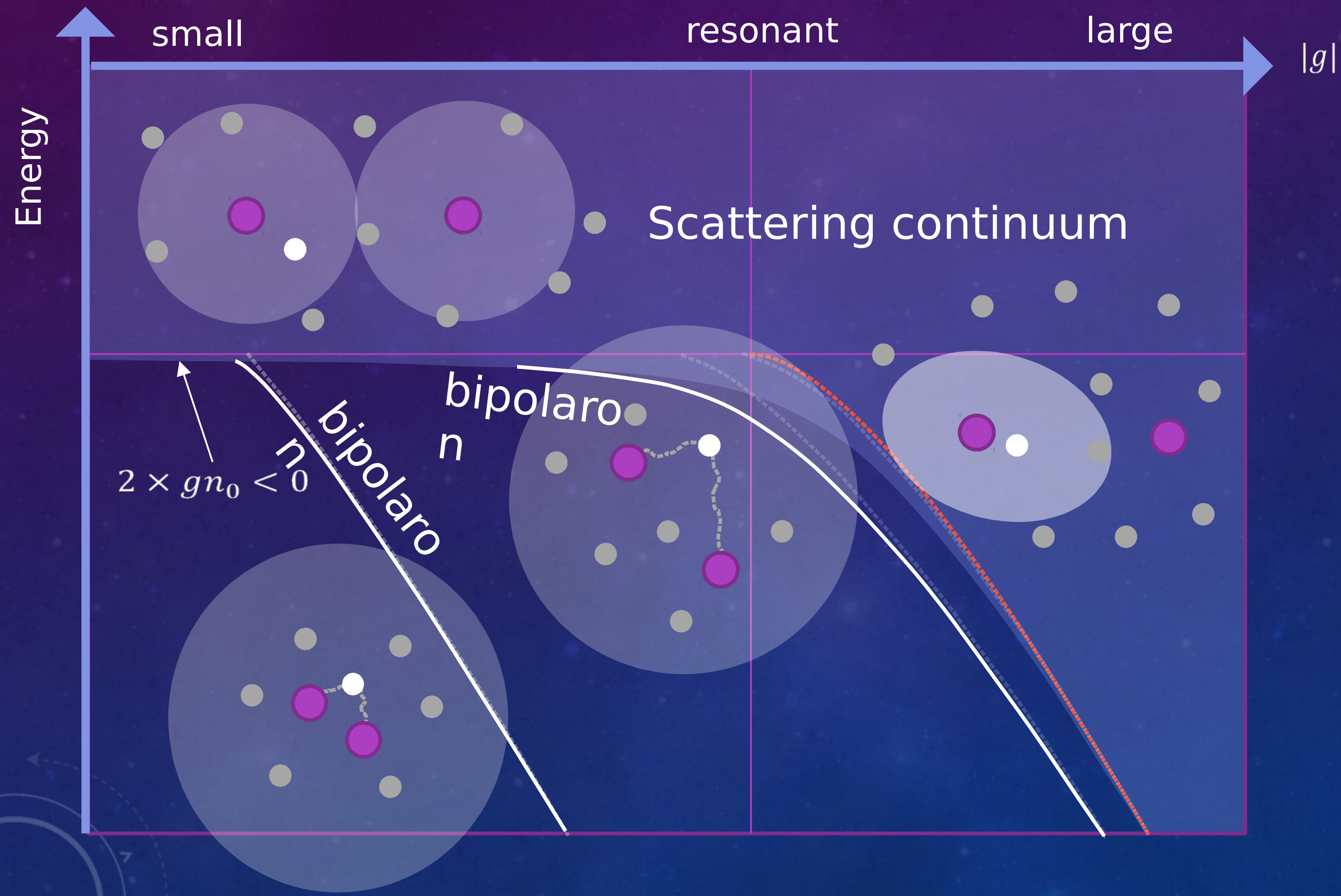
RESULT: ENERGY SPECTRUM



$$\frac{M}{m} = 30$$

$$n_0 a_B^3 = 0.0005$$

RESULT: ENERGY SPECTRUM



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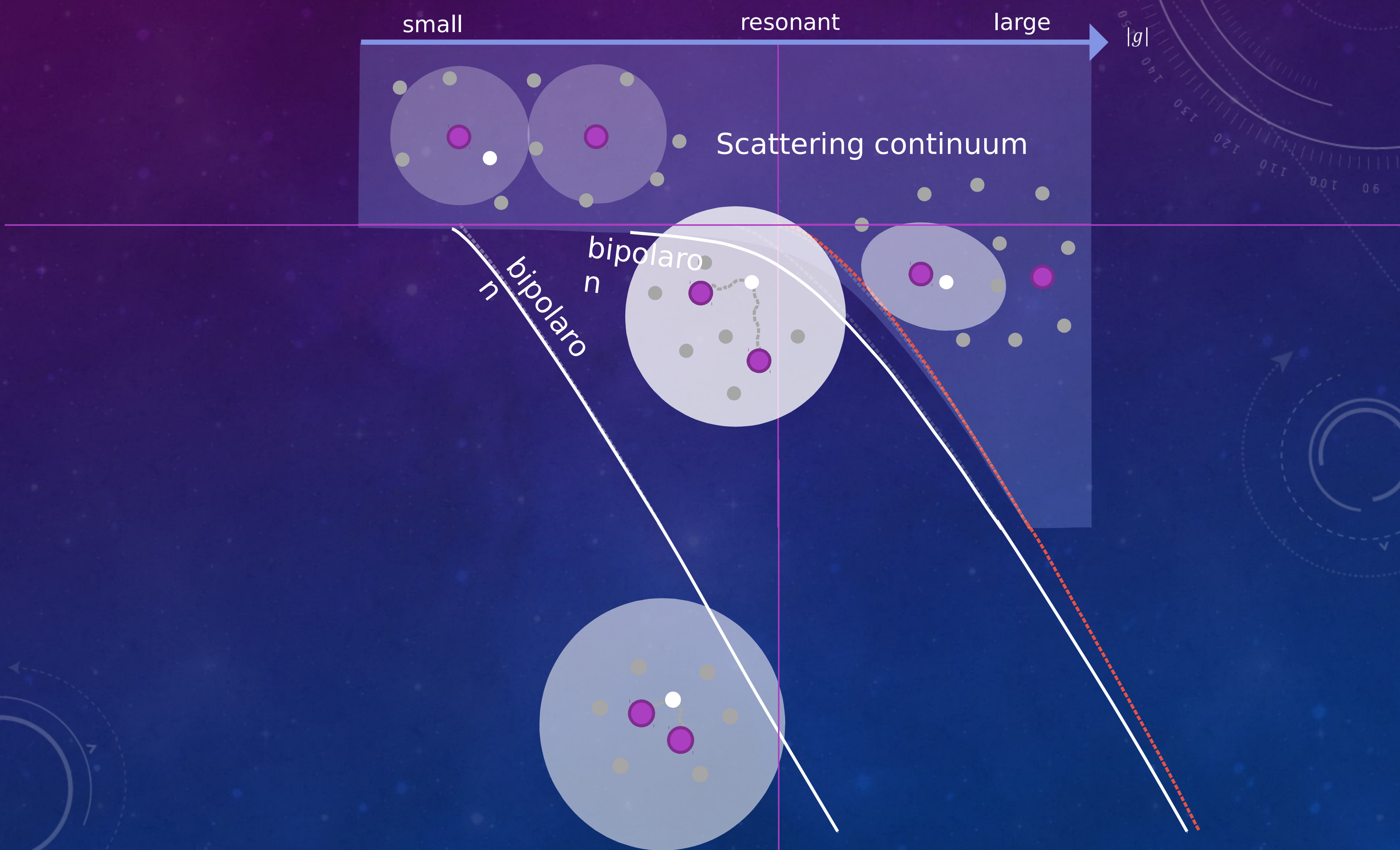

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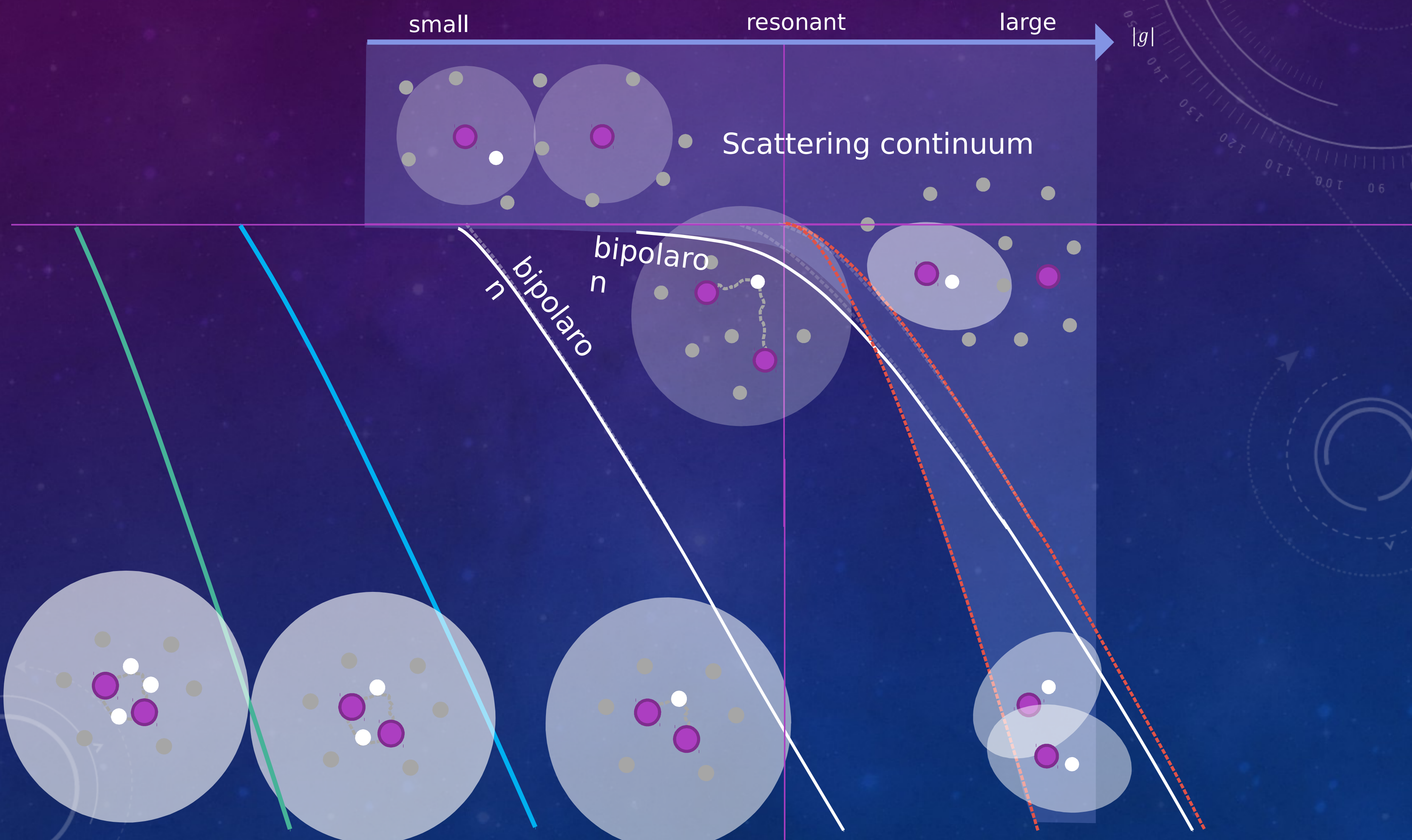

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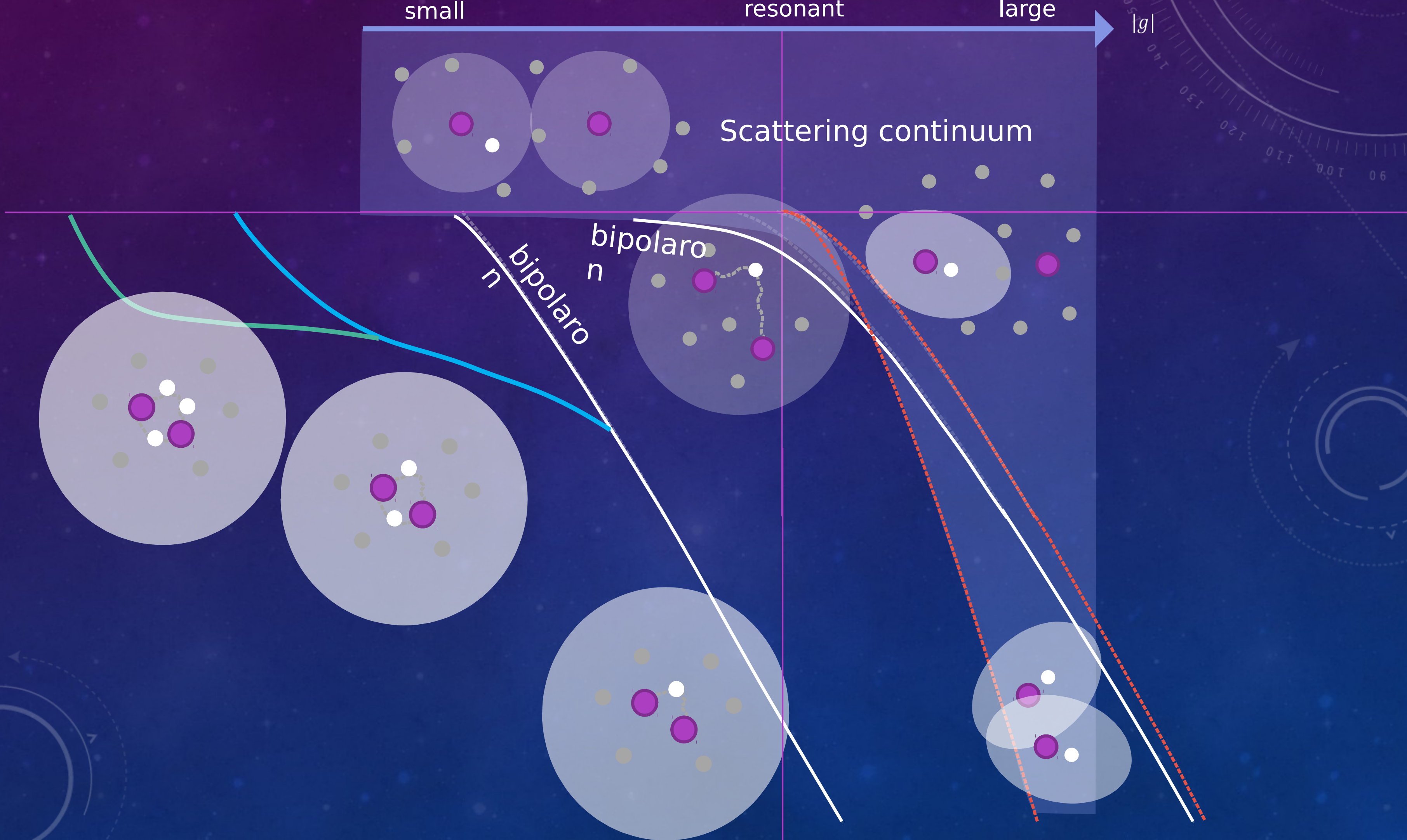

POSSIBLE EXPERIMENTAL OBSERVATIONS

Heavy impurities (e.g. ^{133}Cs) in a condensate of light bosons (e.g. ^{7}Li)

- Polaron RF spectroscopy: mean-field shift due to the polaron interaction
- Loss by recombination: shift of the loss peak with the light boson density

CONCLUSIONS

arXiv:1607.04507

- The interaction between two impurities in a Bose-Einstein goes from a Yukawa attraction to an **Efimov attraction**.
- This attraction can bind the two impurities into one or several **bipolarons**, that asymptote to **Efimov trimers** of two impurities and a boson.
- The crossover region where bipolarons appear constitutes is an interesting few-to-many-body problem.