Disappearance of bubbles in hot ^{28,34}Si nuclei

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Outline

- Bubble structure
- Theoretical models
  - Finite temperature Hartree-Fock (FT)
  - Hartree-Fock plus BCS at finite temperature (FTBCS)
  - Hartree-Fock plus Exact Pairing at finite temperature (FTEP)
- Bubble in hot $^{28,34}Si$ nuclei
- Discussion and conclusion

Bubble Structure

A Spherical Shell Nuclear Model

H. A. Wilson
Rice Institute, Houston, Texas
April 19, 1946

A DISCUSSION of the energies of the beta-, gamma-, and alpha-rays from the naturally radioactive elements led the writer[1] in 1933 to suggest that the nuclei of these elements may have sets of equally spaced energy levels with spacings of 0.387 Mev.

Recently Wiedenbeck[2] has reported equally spaced levels in Au, Ag, In, Cd, and Rh with spacings between 0.36 and 0.50 Mev. Equally spaced levels with separations about 0.4 Mev. are also found with N, Be, Al, and Co. It seems probable therefore that most atomic nuclei have a set of equally spaced levels with separations near to 0.4 Mev.

An account of a simple classical mechanical model of a nucleus, which has nearly equally spaced levels with spacings equal to about 0.4 Mev for all nuclei from beryllium to uranium, follows.

The model is a thin spherical shell which is supposed to be flexible, inextensible, and uniformly charged with positive electricity. The possible frequencies of vibration of the

neutrons was obtained by putting the target area equal to $2\pi r^2$. Classically the target area would be πr^2 so the classical radius would be 14×10^{-13} which is nearly equal to 15×10^{-13}.

The threshold levels for x-ray excitation for Au, In, Cd, Ag, and Rh were measured by Wiedenbeck[2] and found to be nearly equal with an average value of 1.2 Mev. According to the shell model the threshold should be $(10)^{\frac{1}{2}}\Delta E$ which is about 1.3 Mev.

The formation of a shell instead of a sphere may be caused by saturation of the nuclear forces. Thus if we suppose each particle cannot be strongly attracted by more than four nearby particles, it is easy to see that the electrostatic forces should pull out a sphere into a shell.

The area of the shell $S=4\pi r^2$ must be determined by the number N of neutrons and the number Z of protons in it.

H. A. Wilson, Phys. Rev. **69**, 538 (1946)

Volume 46B, number 3 PHYSICS LETTERS 1 October 1973

POSSIBLE BUBBLE NUCLEI - ^{36}Ar AND ^{200}Hg

X. CAMPI* and D.W.L. SPRUNG
Physics Department, McMaster University, Hamilton, Ontario, Canada L8S 4M1

Received 2 August 1973

The question of bubble configurations for ^{36}Ar and ^{200}Hg is studied by Hartree Fock calculations using the density dependent force G-0. By imposing a constraint the solutions can be studied as a function of bubble deformation. Pairing is included in a self consistent manner via a B.C.S. calculation. Factors leading to bubble versus uniform distributions are discussed.

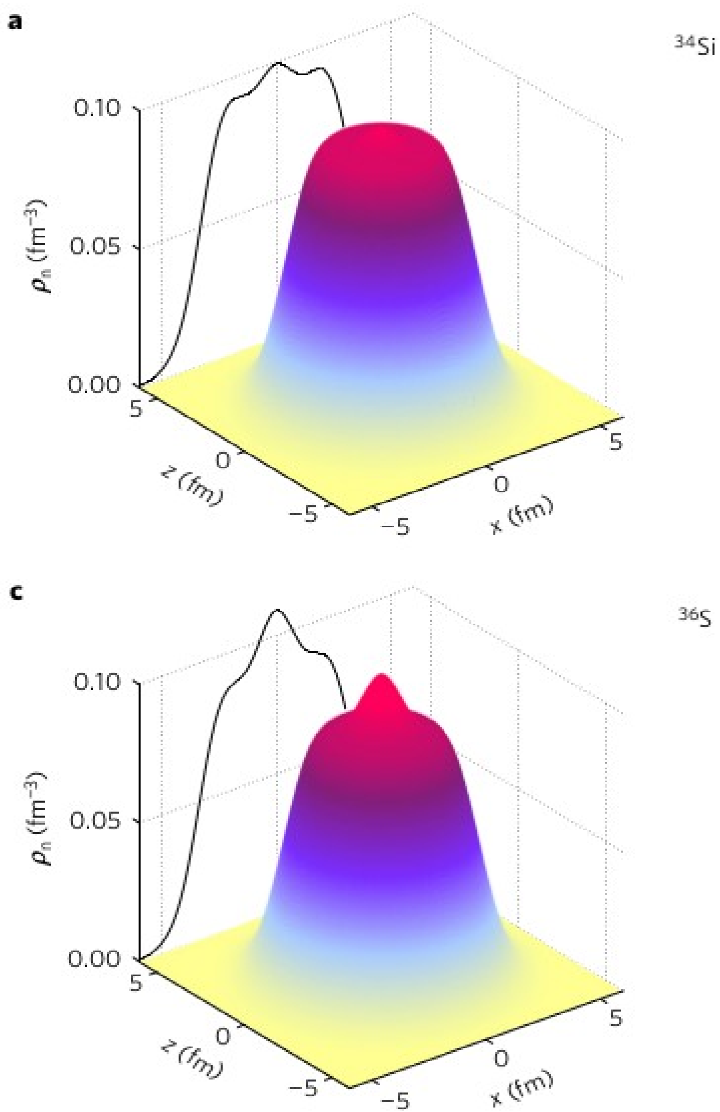
About a year ago, Wong [1] revived interest in the possible existence of bubble nuclei, which are nuclei having a region of anomalously low density in the interior, as opposed to the generally accepted uniform

bound than in the uniform model. If they rise high enough in energy, the highest s-level will empty, hence depleting the central density of particles. Concurrently, the lower s-levels being less bound will increase in

X. Campi and D. W. L. Sprung, Phys. Lett. B**46**, 291 (1973)

Bubble Structure

•Nucleon density of ^{34}Si and ^{36}S*

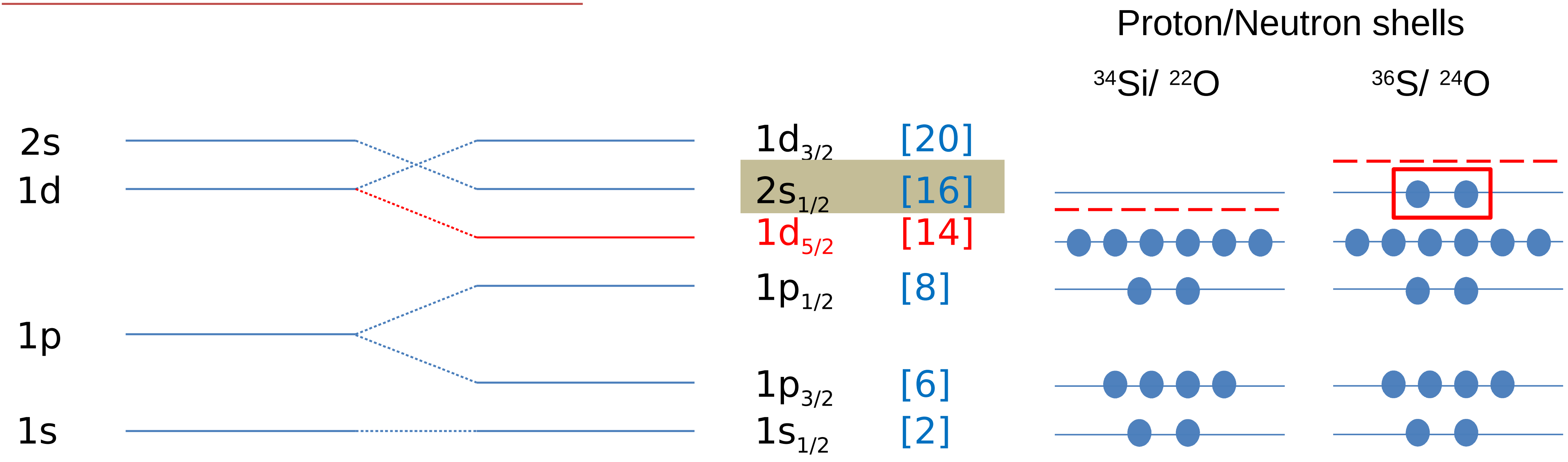


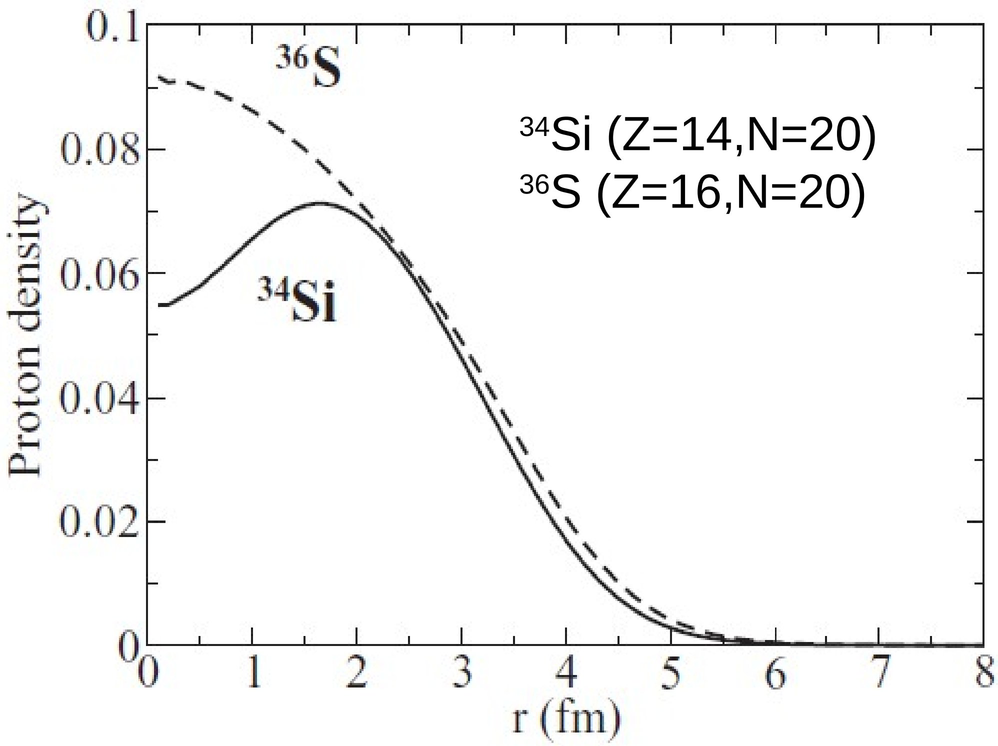
$$F = \frac{\rho_{max} - \rho_{cent}}{\rho_{max}}$$

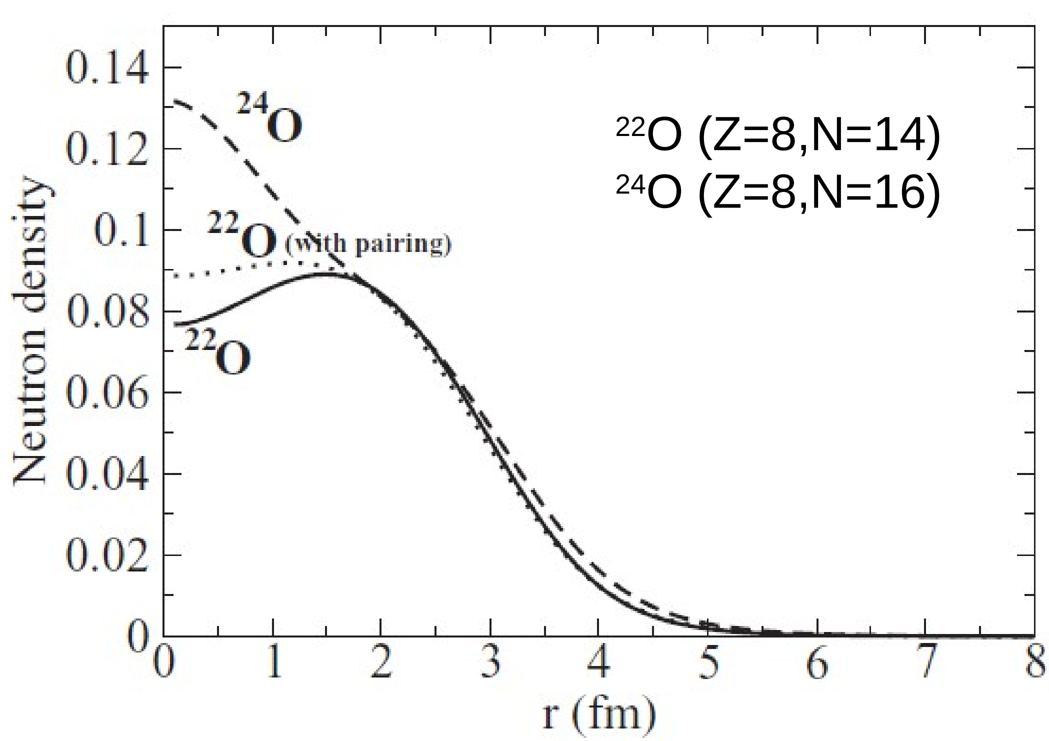
F: depletion factor
ρ_{max} : maximum density
ρ_{cent} : central density

*A. Mutschler *et al*, Nature **13**, 152 (2017)

Bubble structure

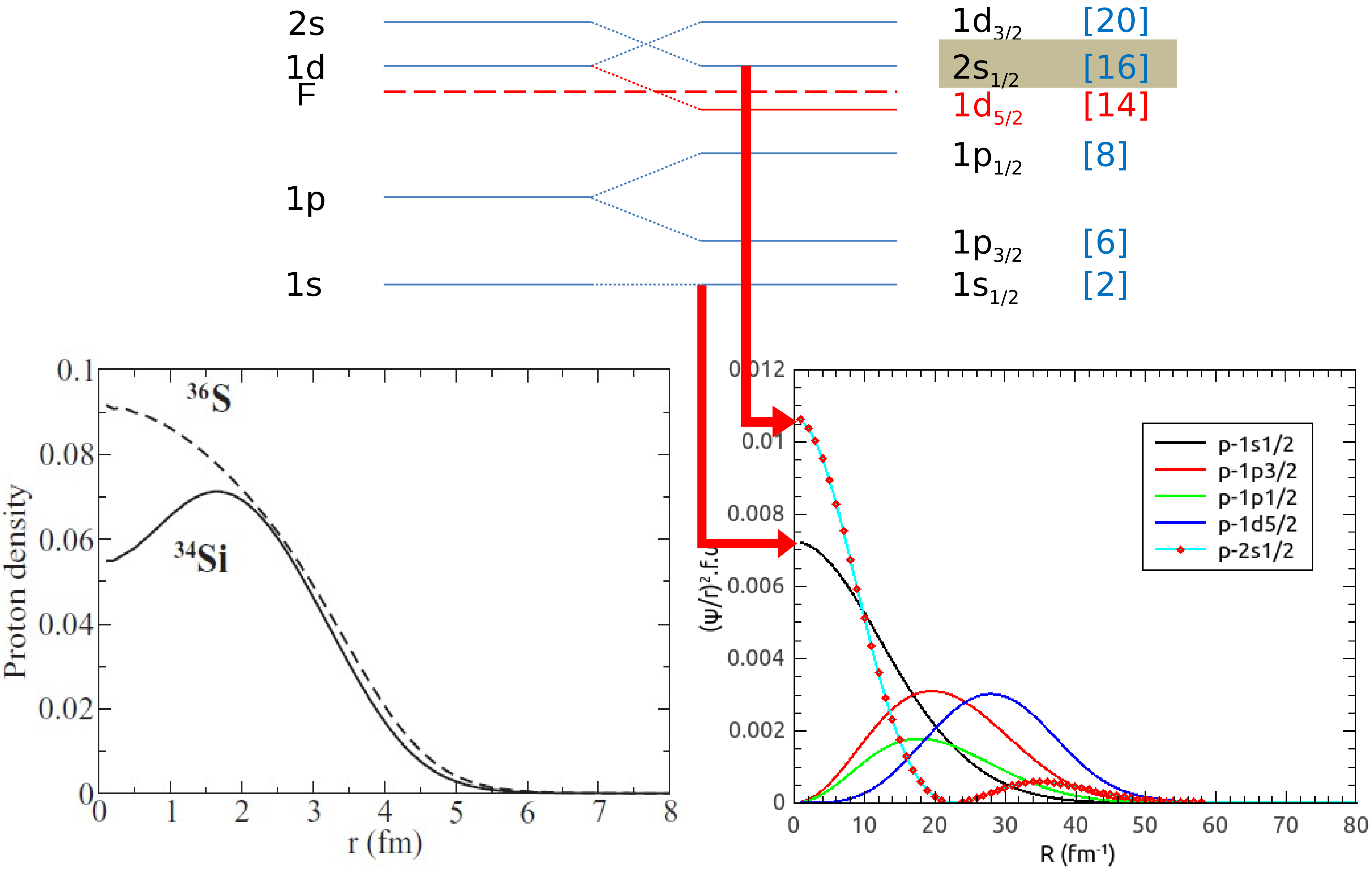






M. Grasso *et al*, Phys. Rev. C **79**, 034318 (2009).

Bubble structure



Bubble Structure

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A proton density bubble in the doubly magic ^{34}Si nucleus

A. Mutschler[1,2], A. Lemasson[2,3], O. Sorlin[2]*, D. Bazin[4], C. Borcea[5], R. Borcea[5], Z. Dombrádi[6], J.-P. Ebran[7], A. Gade[4], H. Iwasaki[4], E. Khan[1], A. Lepailleur[2], F. Recchia[3], T. Roger[2], F. Rotaru[5], D. Sohler[6], M. Stanoiu[5], S. R. Stroberg[4,8], J. A. Tostevin[9], M. Vandebrouck[1], D. Weisshaar[3] and K. Wimmer[3,10,11]

Many properties of the atomic nucleus, such as vibrations, rotations and incompressibility, can be interpreted as due to a two-component quantum liquid of protons and neutrons. Electron scattering measurements on stable nuclei demonstrate that their central densities are saturated, as for liquid drops. In exotic nuclei near the limits of mass and charge, with large imbalances in their proton and neutron numbers, the possibility of a depleted central density, or a 'bubble' structure, has been discussed in a recurrent manner since the 1970s. Here we report first experimental evidence that points to a depletion of the central density of protons in the short-lived nucleus ^{34}Si. The proton-to-neutron density asymmetry in ^{34}Si offers the possibility to place constraints on the density and isospin dependence of the spin-orbit force—on which nuclear models have disagreed for decades—and on its stabilizing effect towards limits of nuclear existence.

*A. Mutschler *et al*, Nature **13**, 152 (2017)

$2s_{1/2}$ occupancy=0.17

Finite temperature Hartree-Fock

- Mean-field description

$$[\boldsymbol{T} + \boldsymbol{V_D}]\boldsymbol{\psi_i} + \boldsymbol{V_{Ex}}\boldsymbol{\psi_j} = \boldsymbol{E_i}\boldsymbol{\psi_i}$$

Skyrme interaction*

$$V = \sum_{i<j} V_{ij}^{(2)} + \sum_{i<j<k} V_{ijk}^{(3)}$$

$$V_{ij}^{(2)} = t_0(1 + x_0 P_\sigma)\delta(\vec{r}) + \frac{1}{2} t_1 \left[\delta(\vec{r})\vec{k}^2 + \overrightarrow{k'}^2 \delta(\vec{r})\right] + t_2 \overrightarrow{k'} \delta(\vec{r})\vec{k} + iW_0\left(\vec{\sigma}_i + \vec{\sigma}_j\right)\vec{k} \times \delta(\vec{r})\vec{k}$$

$$V_{ijk}^{(3)} = t_3 \delta\left(\vec{r}_i - \vec{r}_j\right)\delta\left(\vec{r}_j - \vec{r}_k\right)$$

*T. H. R. Skyrme, Nucl. Phys **9** (1959).

Skyrme interaction

Vautherin and Brink* showed that three – body interaction is equivalent to two – body interaction and they depend on nucleon density :

$$v_{ijk}^{(3)} \rightarrow v_{ij}^{(2)} = \frac{t_3}{6}(1+\hat{P}_\sigma)\delta(\vec{r}_1-\vec{r}_2)\rho^{\alpha}\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)$$

* D. Vautherin & D. M. Brink, Phys. Rev. C **5**, 1972

The Skyrme interaction nowadays:

$$V(\vec{r}_1,\vec{r}_2) = t_0(1+x_0\hat{P}_\sigma)\delta(\vec{r}_1-\vec{r}_2)$$

Central potential

$$+\frac{1}{2}t_1(1+x_1\hat{P}_\sigma)[\vec{k}'^2\delta(\vec{r})+\delta(\vec{r})\vec{k}^2]+t_2(1+x_2\hat{P}_\sigma)\vec{k}'^2.\delta(\vec{r})\vec{k}^2$$

Non – local potential

$$+iW(\vec{\sigma}_1+\vec{\sigma}_2)\vec{k}'\times\delta(\vec{r}_1-\vec{r}_2)\vec{k}$$

Spin – orbit potential

$$+\frac{t_3}{6}(1+x_3\hat{P}_\sigma)\delta(\vec{r}_1-\vec{r}_2)\rho^{\alpha}\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)$$

Density dependent potential

Where: $k = \frac{1}{2i}(\nabla_1-\nabla_2)$ and k' is the conjugate of k.

Total nucleon density $\rho = \rho_n + \rho_p$

Spin – exchange operator $P^{\sigma} = \frac{1}{2}(1+\vec{\sigma}_1\vec{\sigma}_2)$

Skyrme Hartree-Fock

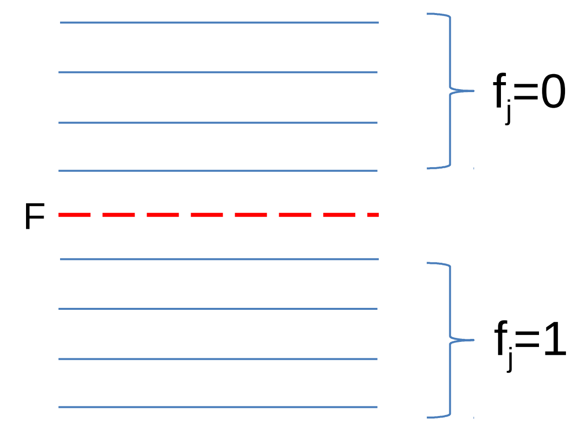
Energy

$$E = \langle \Phi | T + V | \Phi \rangle$$
$$= \sum_{i=1}^{A} \left\langle i \left| \frac{P_i^2}{2m} \right| i \right\rangle + \frac{1}{2} \sum_{i,j=1}^{A} \left\langle ij \left| V_{ij}^{(2)} \right| ij \right\rangle + \frac{1}{6} \sum_{i,j,k=1}^{A} \left\langle ijk \left| V_{ijk}^{(2)} \right| ijk \right\rangle$$

T=0

Densities

$$\rho(\boldsymbol{r}) = \sum_{j,s,t} f_j \left| \varphi_j(\boldsymbol{r}, s, t) \right|^2$$



Kinetic energy densities

$$\tau(\boldsymbol{r}) = \sum_{j,s,t} f_j \left| \nabla \varphi_j(\boldsymbol{r}, s, t) \right|^2$$

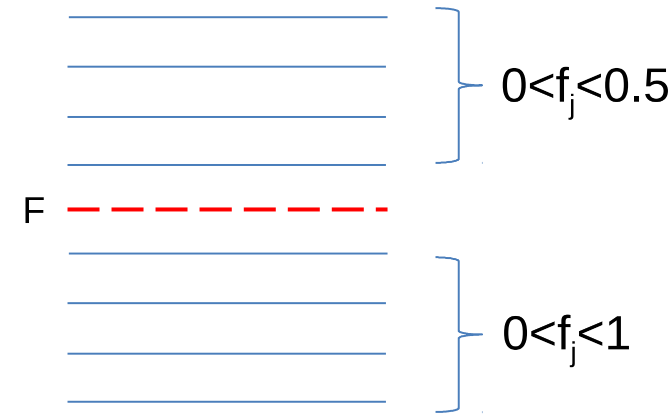
Spin-orbit densities

$$J(\boldsymbol{r}) = (-i) \sum_{i,s,s',t} f_j \varphi_j^*(\boldsymbol{r}, s, t) \left[\nabla \varphi_j(\boldsymbol{r}, s', t) \times \langle \sigma_s | \hat{\boldsymbol{\sigma}} | \sigma_{s'} \rangle \right]$$

T≠0

Densities

$$\rho(\mathbf{r}) = \sum_{j,s,t} f_j \left|\varphi_j(\mathbf{r},s,t)\right|^2$$



Kinetic energy densities

$$\tau(\mathbf{r}) = \sum_{j,s,t} f_j \left|\nabla\varphi_j(\mathbf{r},s,t)\right|^2$$

Spin-orbit densities

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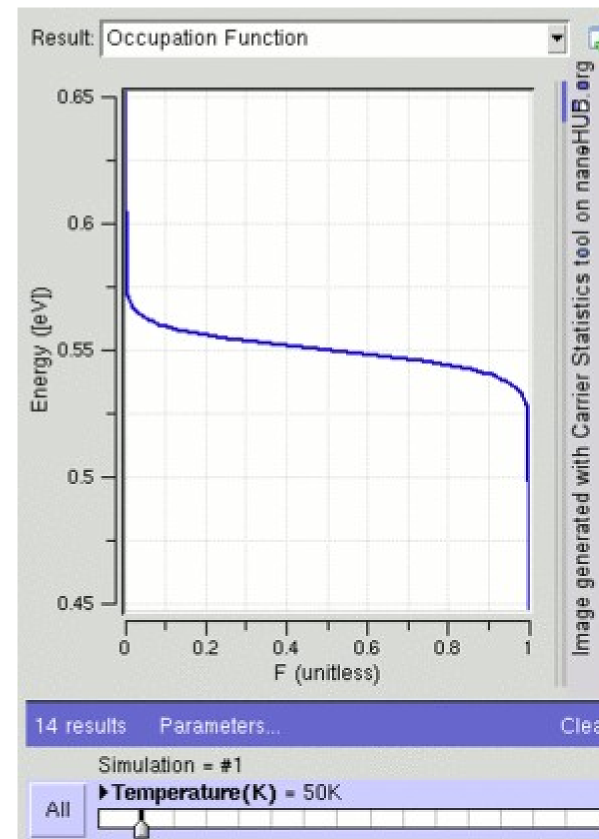
Occupation number

$$f_j = \frac{1}{e^{(\epsilon_j-\lambda)\beta} + 1}$$

Nucleon densities

$$\rho_q(r) = 2\sum_j \frac{f_j \Omega_j R_j^2(r)}{4\pi r^2}$$

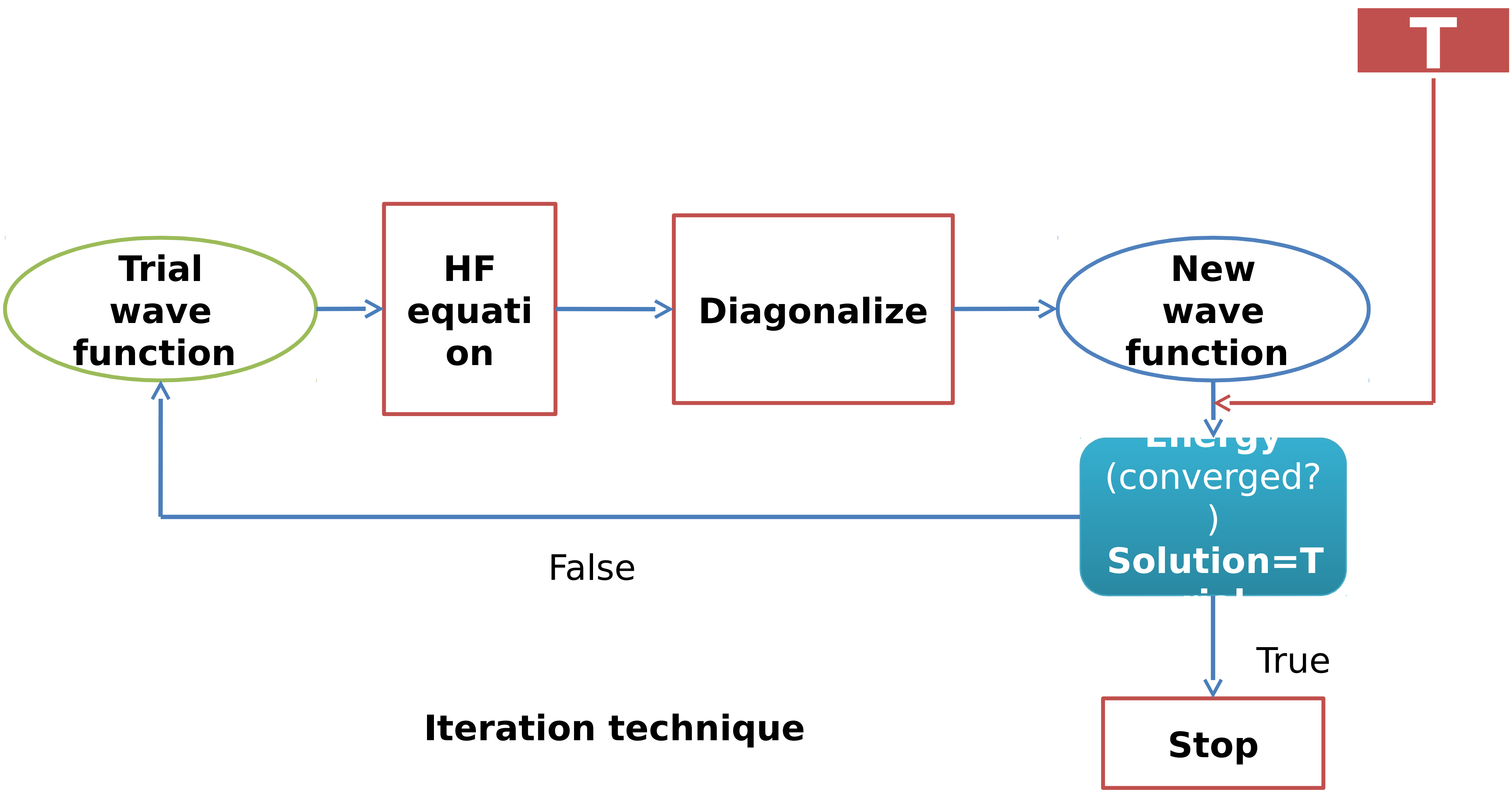


f_j : occupation number
ϵ_j : single-particle energy
$\beta=1/T$
λ: chemical potential
$\Omega_j=j+1/2$
$\Phi_j(r)=R_j/r$: radial wave function

Finite temperature Hartree-Fock

BSk17*	t_0	t_1	t_2	t_3	x_0	x_1	x_2	x_3	W_0	α	J^2
Values	-1837.33	389.102	-3.1742	11523.8	0.411377	-0.832102	49.4875	0.654962	145.885	0.3	1

* S Goriely *et al*, Phys. Rev. Lett. 102 (**24**), 242501 (2009)



Finite temperature BCS

Pairing gap

$$\Delta_{BCS} = \Omega_j G \sum_j u_j v_j (1 - 2n_j)$$

Particle number

$$N = 2 \sum_j \Omega_j f_j$$

BCS equations

Quasi-occupation number

$$n_j = \frac{1}{e^{\beta E_j} + 1}, \qquad \beta = \frac{1}{T}$$

Quasi-particle energy

$$E_j = \sqrt{(\epsilon_j - \lambda)^2 + \Delta_{BCS}^2}$$

Occupation number

$$f_j = u_j^2 n_j + v_j^2 (1 - n_j)$$

Where:

$$u_j^2 = \frac{1}{2}\left(1 + \frac{\epsilon_j - \lambda}{E_j}\right), \qquad v_j^2 = 1 - u_j^2$$

Exact pairing (EP)

$$H = \sum_{j\,m} \epsilon_j a^\dagger_{j\,m} a_{j\,m} + \frac{1}{4} \sum_{j,j'} G_{j\,j'} \sum_{m,m'} a^\dagger_{j\,m} \tilde{a}^\dagger_{j\,m} \tilde{a}_{j'\,m'} a_{j'\,m'} \,, \quad \tilde{a}_{j\,m} \equiv (-1)^{j-m} a_{j\,-m}$$

where

$$L^-_j = \frac{1}{2} \sum_m \tilde{a}_{j\,m} a_{j\,m} \,, \quad L^+_j = \left(L^-_j\right)^\dagger = \frac{1}{2} \sum_m a^\dagger_{j\,m} \tilde{a}^\dagger_{j\,m}$$

$$L^z_j = \frac{1}{2} \sum_m \left(a^\dagger_{j\,m} a_{j\,m} - \frac{1}{2} \right) = \frac{1}{2}(N_j - \Omega_j)$$ and $$\Omega_j = (2j+1)/2$$

$$H = \sum_j \epsilon_j \Omega_j + 2 \sum_j \epsilon_j L^z_j + \sum_{j\,j'} G_{j\,j'} L^+_j L^-_{j'}$$

Diagonal elements

$$\langle \{s_j\}, \{N_j\} | H | \{s_j\}, \{N_j\} \rangle = \sum_j \left(\epsilon_j N_j + \frac{G_{j\,j}}{4} (N_j - s_j)(2\Omega_j - s_j - N_j + 2) \right)$$

Off – diagonal elements

$$\langle \{s_j\}, \ldots N_j + 2, \ldots N_{j'} - 2, \ldots | H | \{s_j\}, \ldots N_j, \cdots N_{j'}, \ldots \rangle$$

$$= \frac{G_{j\,j'}}{4} \sqrt{(N_{j'} - s_{j'})(2\Omega_{j'} - s_{j'} - N_{j'} + 2)(2\Omega_j - s_j - N_j)(N_j - s_j + 2)}$$

Exact pairing at finite temperature

Partition function

$$Z_{ex} = \sum_s d^{(s)} e^{-E_{ex}^{(s)}\beta}$$

Pairing energy

$$E_{ex} = \frac{1}{Z_{ex}} \sum_s d^{(s)} E_{ex}^{(s)} e^{-E_{ex}^{(s)}\beta}$$

Pairing gap

$$\Delta_{ex} = \sqrt{-G(E_{ex} - E_0)}$$

where

$$E_0 = \sum_j (2\epsilon_j f_j - G f_j^2)$$

Occupation number

$$f_j = \frac{\sum_s d^{(s)} F_j^{(s)} e^{-E_{ex}^{(s)}\beta}}{Z_{ex}}$$

$d^{(s)}$ is the degeneracy of basic states.

G: pairing strength.

$E_{ex}^{(s)}$ are the eigenvalues of EP Hamiltonian.

$F_j^{(s)}$ are the EP occupation number.

FTHF

$$f_j = \frac{1}{e^{(\epsilon_j - \lambda)\beta} + 1}$$

FTBCS

$$f_j = u_j^2 n_j + v_j^2 (1 - n_j)$$

FP

$$f_j = \frac{\sum_s d^{(s)} F_j^{(s)} e^{-E_{ex}^{(s)}\beta}}{Z_{ex}}$$

Exp*

$$f_{j=2s_{1/2}}^{exp} = 0.085$$

Nucleon densities

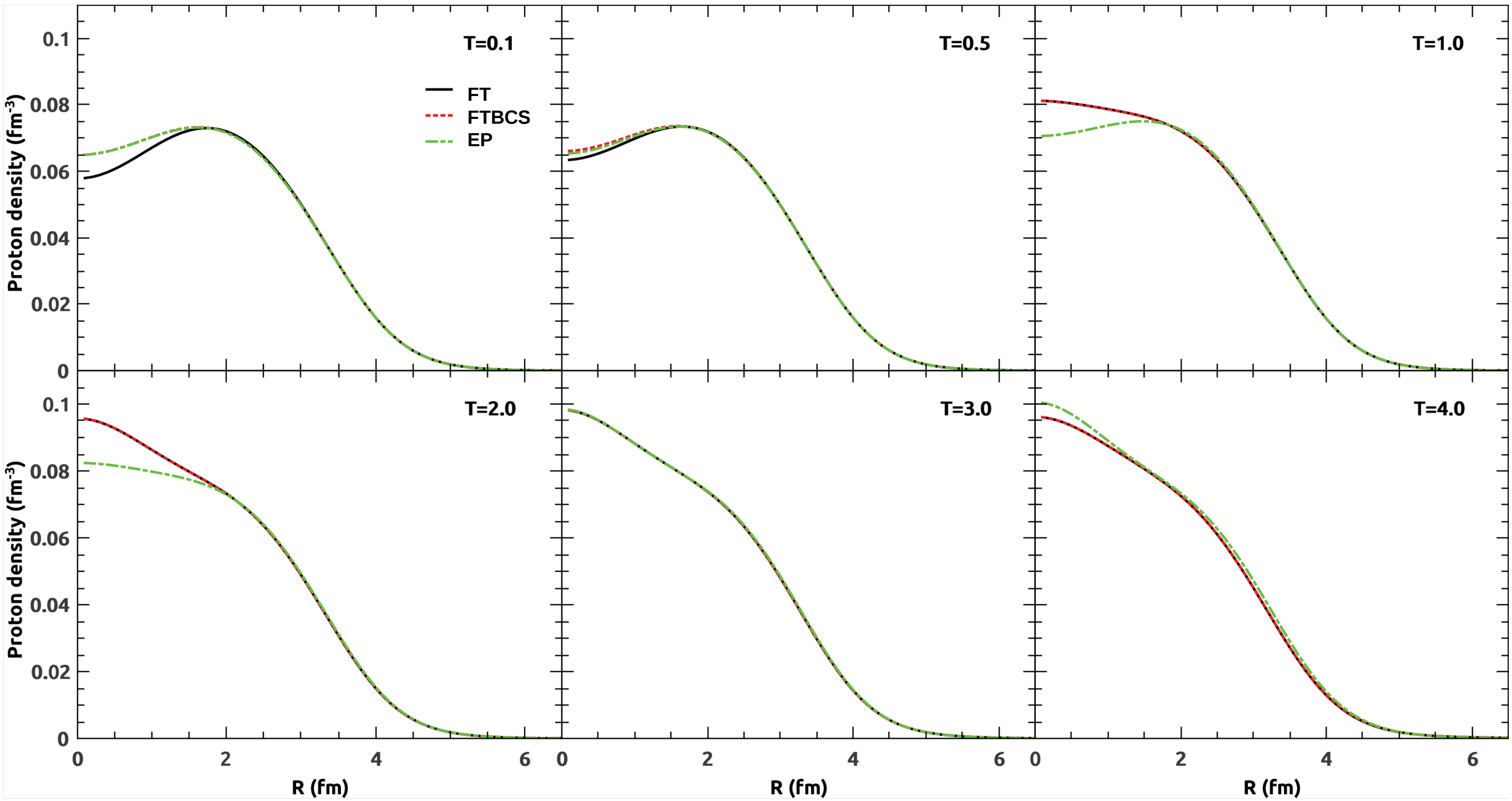
$$\rho_q(r) = 2 \sum_j \frac{f_j \Omega_j R_j^2(r)}{4\pi r^2}$$

*A. Mutschler *et al*, Nature **13**, 152 (2017)

Results

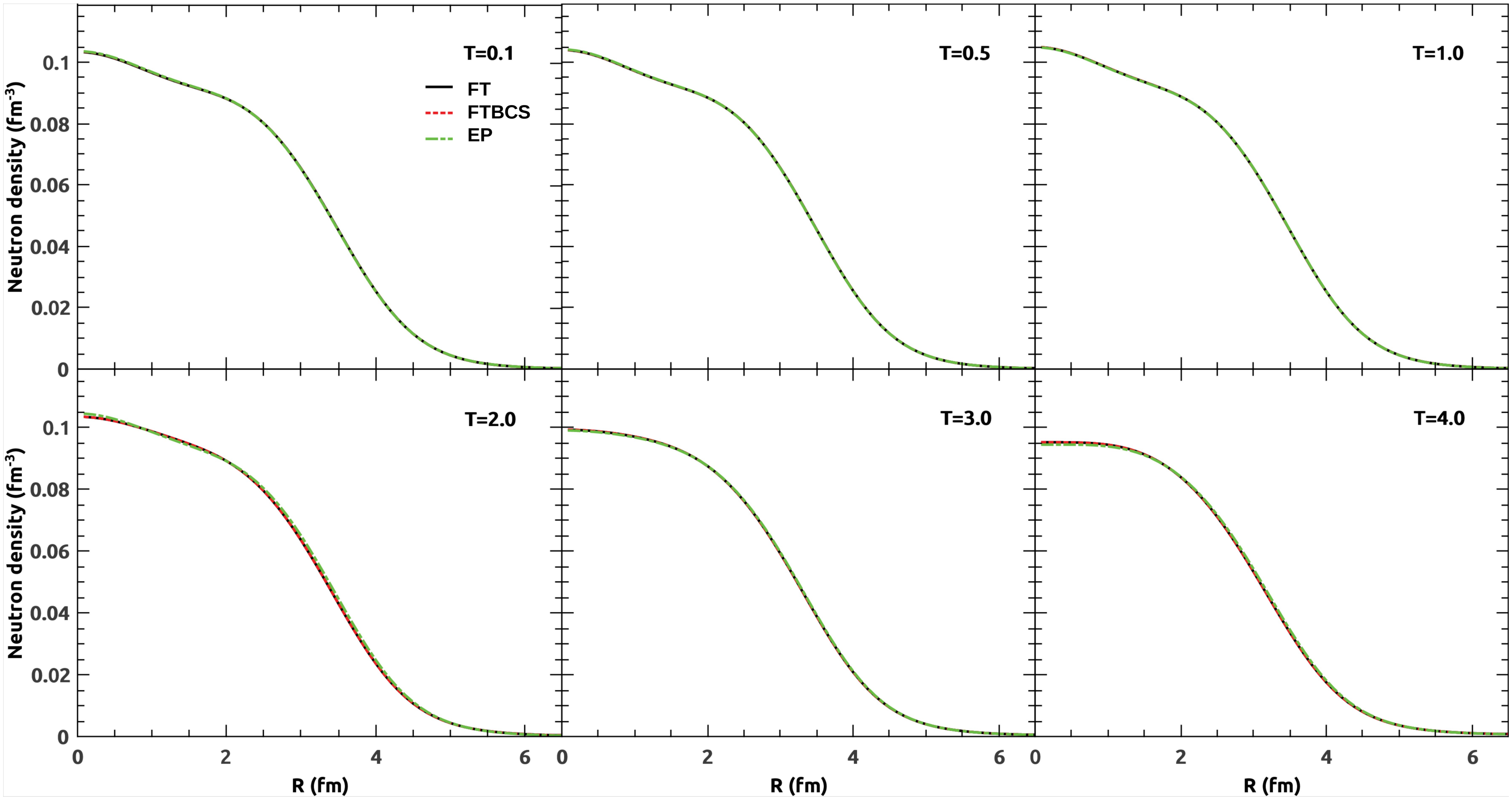
Bubble in ^{34}Si

Proton density



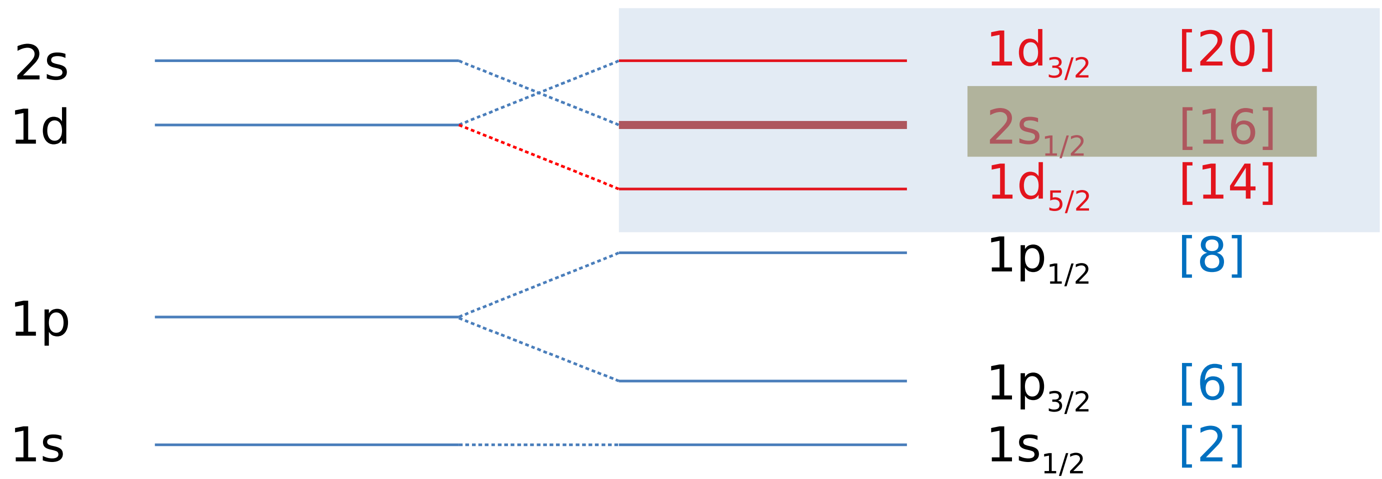
Bubble in ^{34}Si

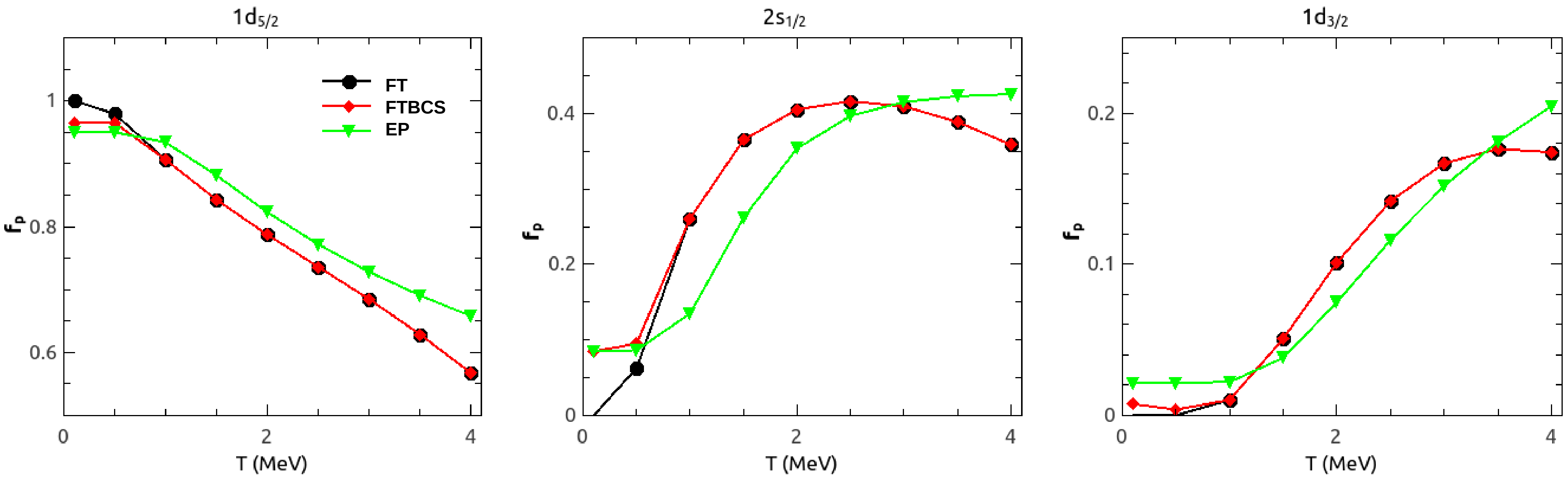
Neutron density



Bubble in ^{34}Si

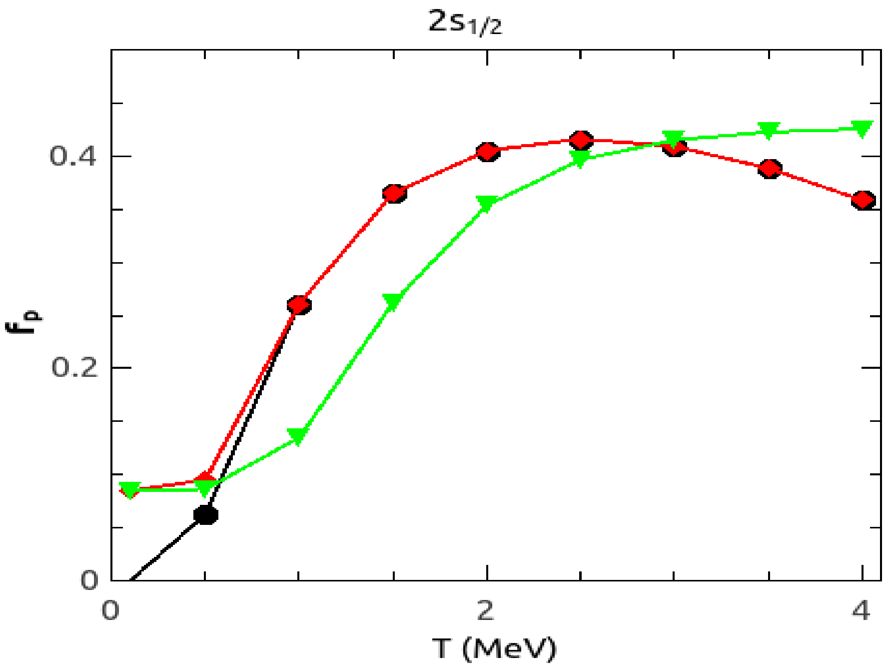
Occupation number

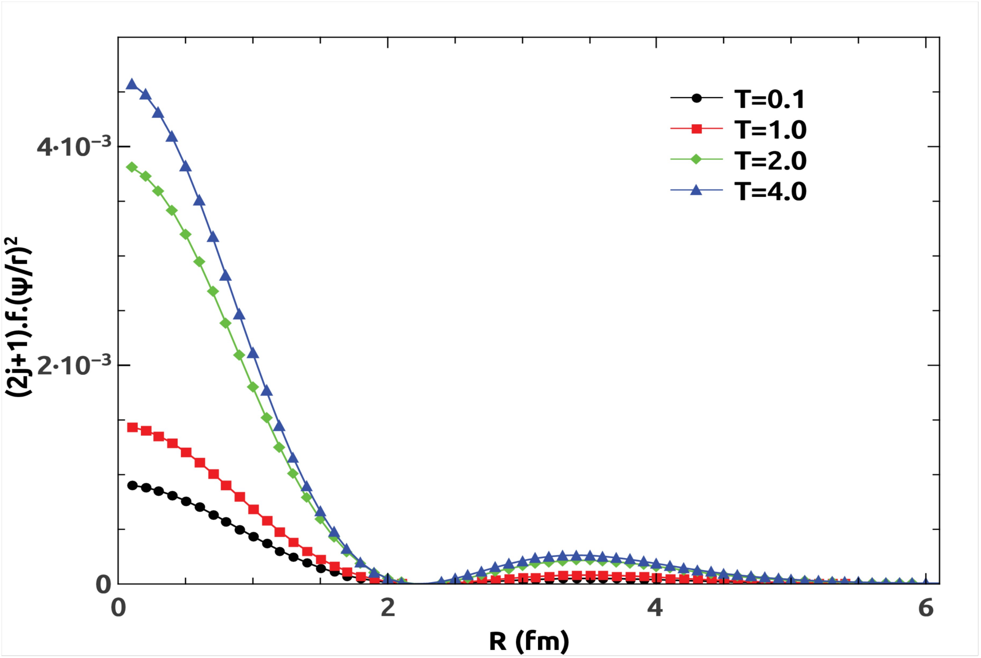


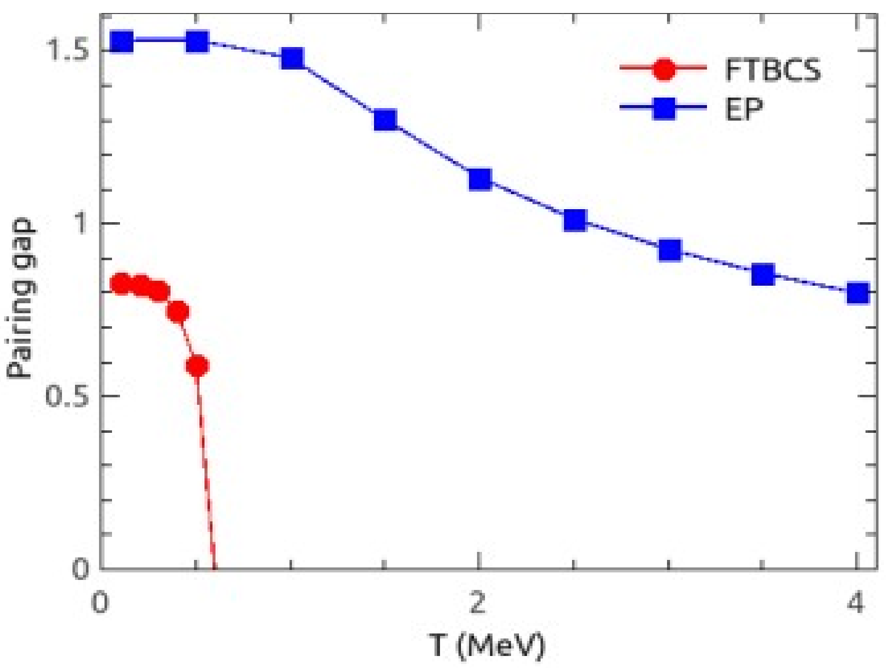


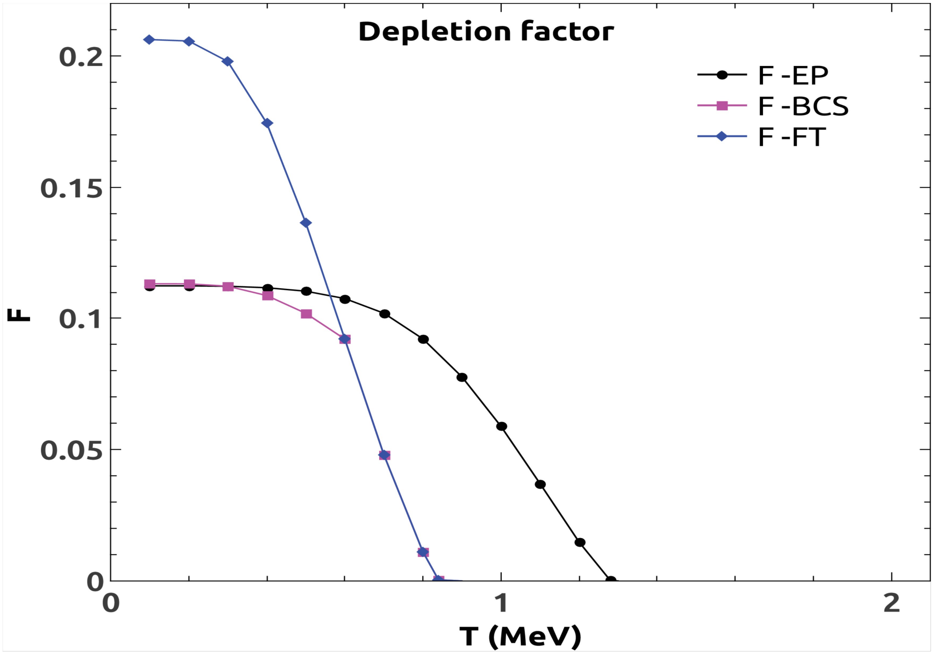
Bubble in ^{34}Si

The contribution of $2s_{1/2}$ orbital wave function



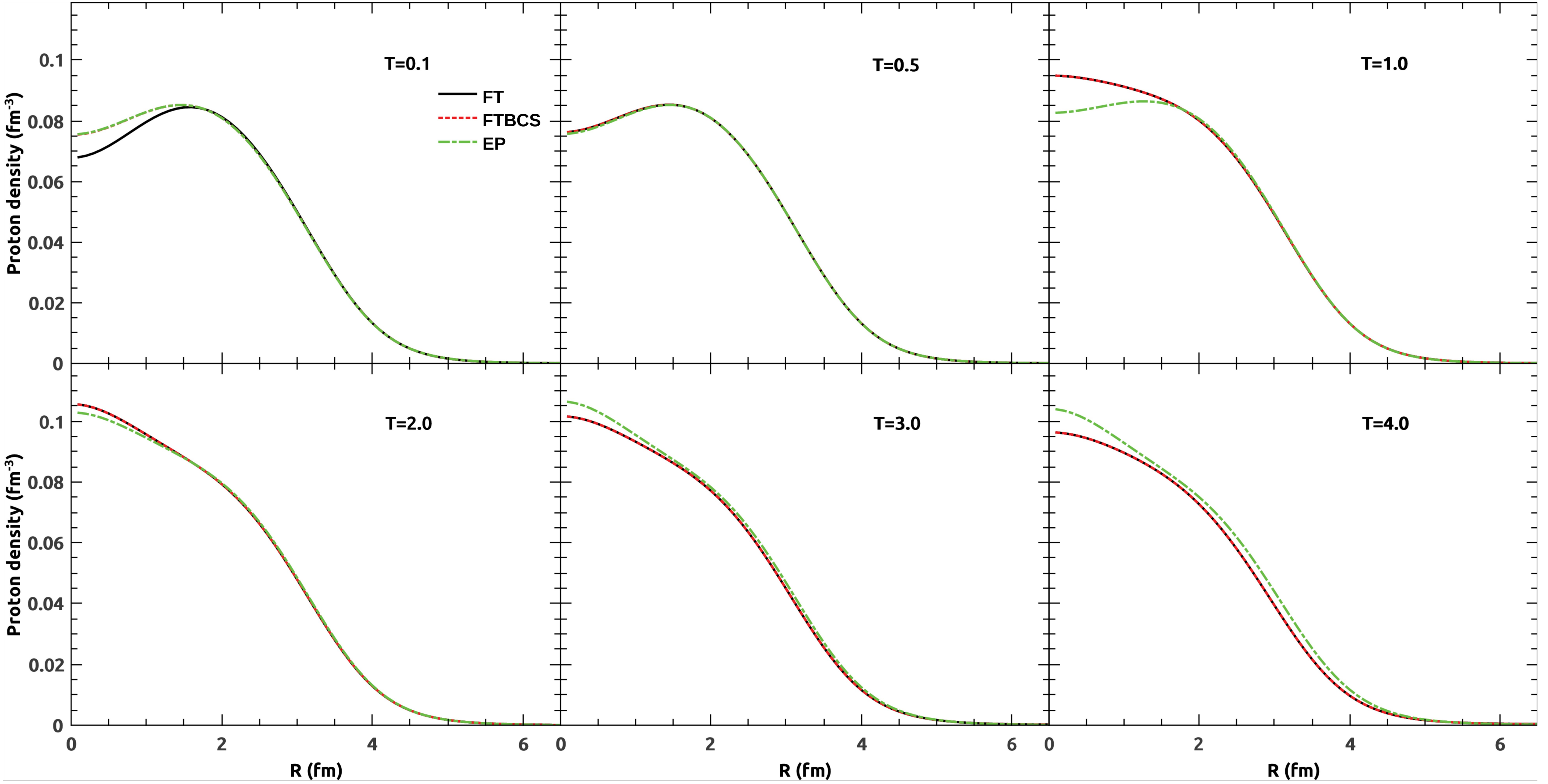






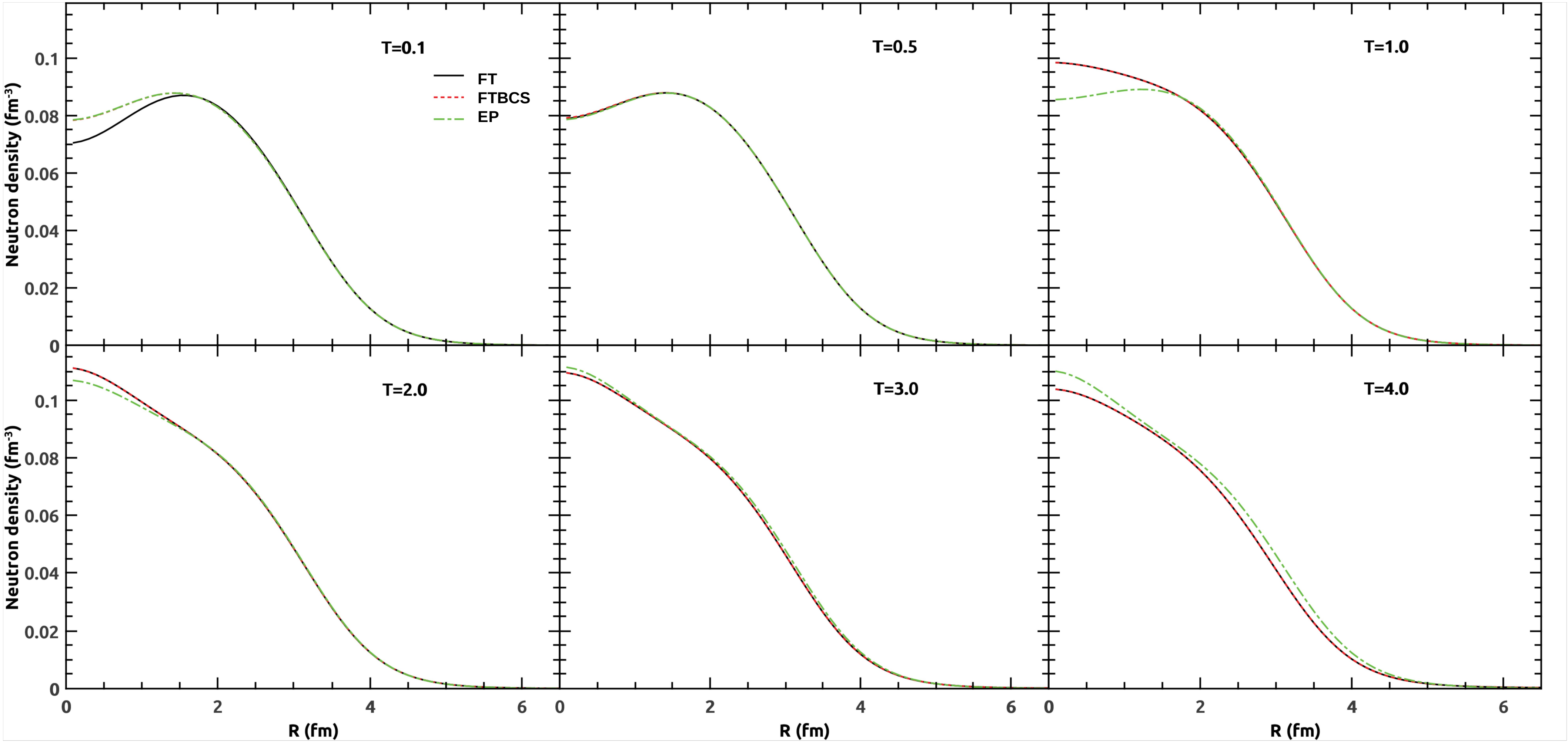
Bubble in ^{28}Si

Proton density



Bubble in ^{28}Si

Neutron density



Conclusion

- The bubble structures of $^{28,34}Si$ are investigated at finite temperature by using three theoretical models: FTHF, FTBCS and EP.
- The bubbles in these nuclei, which are caused by very low s-orbitals occupancies, exit at T=0, reduce with increasing T and disappear when T≥2 MeV.
- The pairing (FTBCS and EP) models predict the bubble structure shallower than non-pairing (FTHF) model at T=0. The bubble structure, which is predicted by EP, disappear slower than FTHF and FTBCS prediction at finite temperature.
- The increasing of $2s_{1/2}$ occupancy ($2f_{j=2s1/2} = 0,17$ at $T = 0$) in $^{28,34}Si$ is the main reason which quenches the bubble with increasing temperature. The bubbles disappear at $T \geq 2$ MeV $\rightarrow f_j \geq 0.4$.

Thank you for your attention

Hoi An Ancient Town
Artist: Huu Duc-2016