Strange nuclear physics from QCD on lattice

Takashi Inoue @Nihon Univ.

for

HAL QCD Collaboration

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QMBP, Danang, Mar 9 2017

Introduction

- Nuclear physics
  - Theories have been developed extensively from 1930's
    - Liquid-drop model and semi-empirical mass formula.
    - Shell models supported by mean-field and Brueckner theory.
    - Variational methods w/ advanced technique for light nuclei.
    - Several sophisticated theories for heavy nuclei in these days,
      - eg. the coupled cluster theory, self consistent greens function method etc.
  - Properties of nuclei are explained and even predicted.

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No free parameter almost

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- Quantum Chromodynamics

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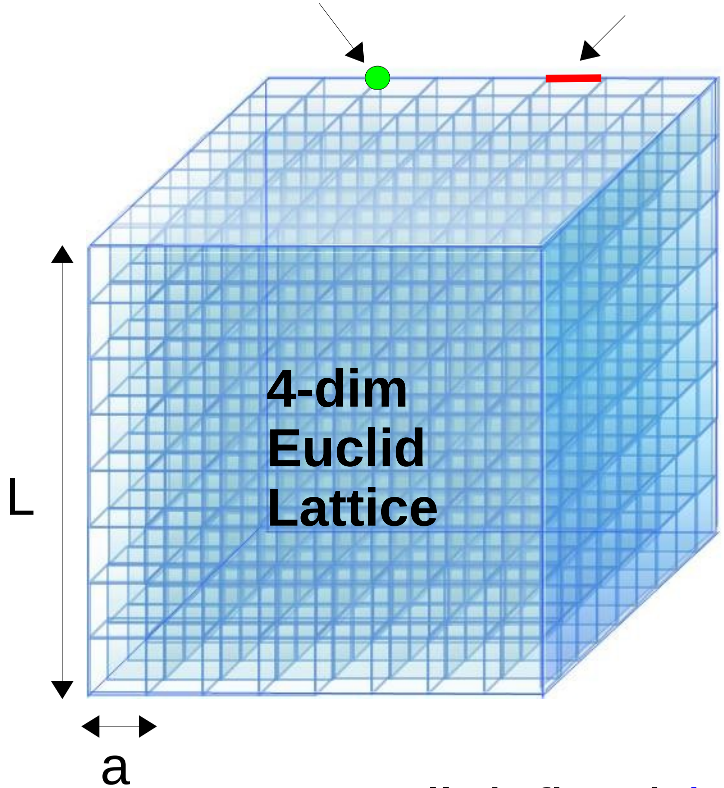
  - is the fundamental theory of the strong interaction,
  - must explain everything, e.g. hadron spectrum, mass of nuclei.
  - But, that is difficult due to the non-perturbative nature of QCD.

Lattice QCD

$$L = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \bar{q}\,\gamma^\mu \left(i\,\partial_\mu - g\,t^a A^a_\mu \right) q - m\,\bar{q}\,q$$

quarks q
on the sites

gluons $U = e^{i a A_\mu}$
on the links



Vacuum expectation value

$$\langle O(\bar{q}, q, U) \rangle$$

path integral

$$= \int dU\, d\bar{q}\, dq\, e^{-S(\bar{q}, q, U)}\, O(\bar{q}, q, U)$$

$$= \int dU\, \det D(U)\, e^{-S_U(U)}\, O(D^{-1}(U))$$

quark propagator

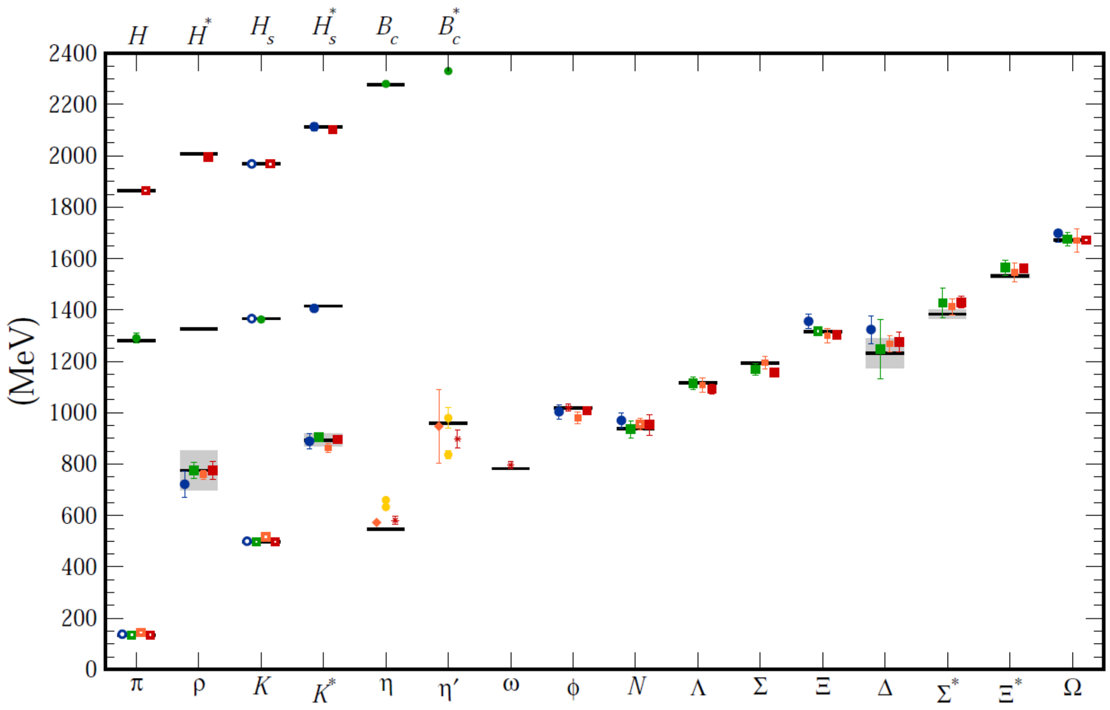
$$= \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i))$$

{ U_i } : ensemble of gauge conf. U
generated w/ probability $\det D(U)\, e^{-S_U(U)}$

- ✯ Well defined (regularized)
- ✯ Manifest gauge invariance
- ✯ Fully non-perturvative
- ✯ Highly predictive

Lattice QCD

- LQCD simulations w/ the physical quark ware done.
  - PACS-CS, Phys. Rev. D81 (2010) 074503
  - BMW, JHEP 1108 (2011) 148



Summary by Kronfeld, arXive 1203.1204

- Mass of (ground state) hadrons are well reproduced!

Lattice QCD

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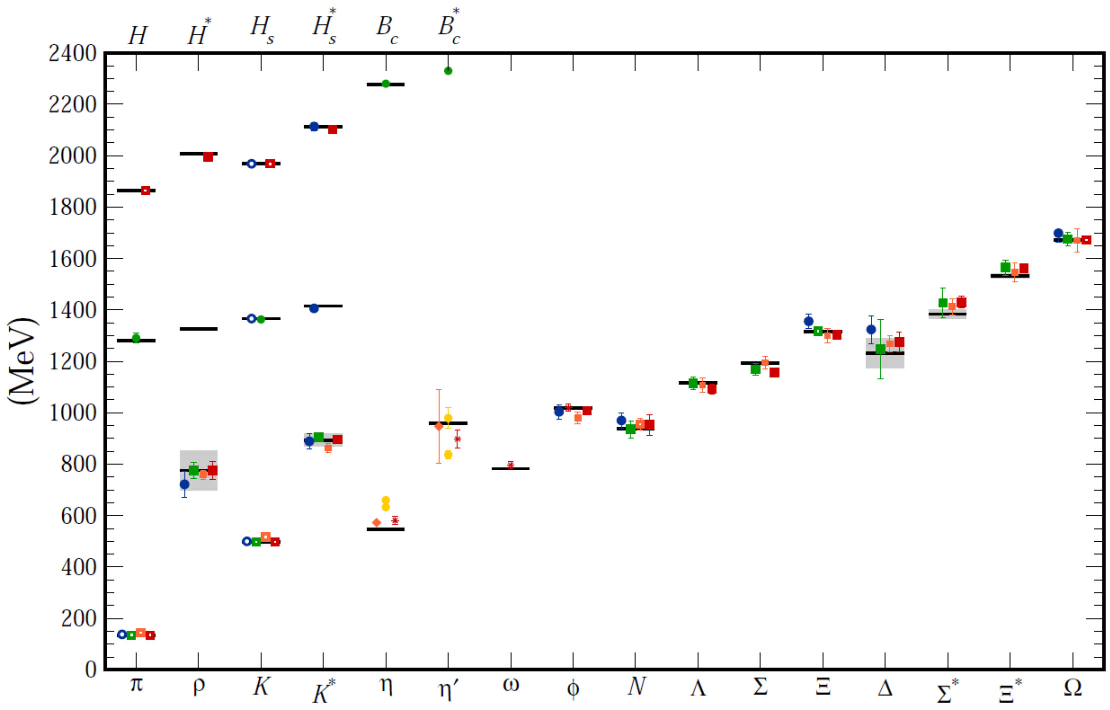
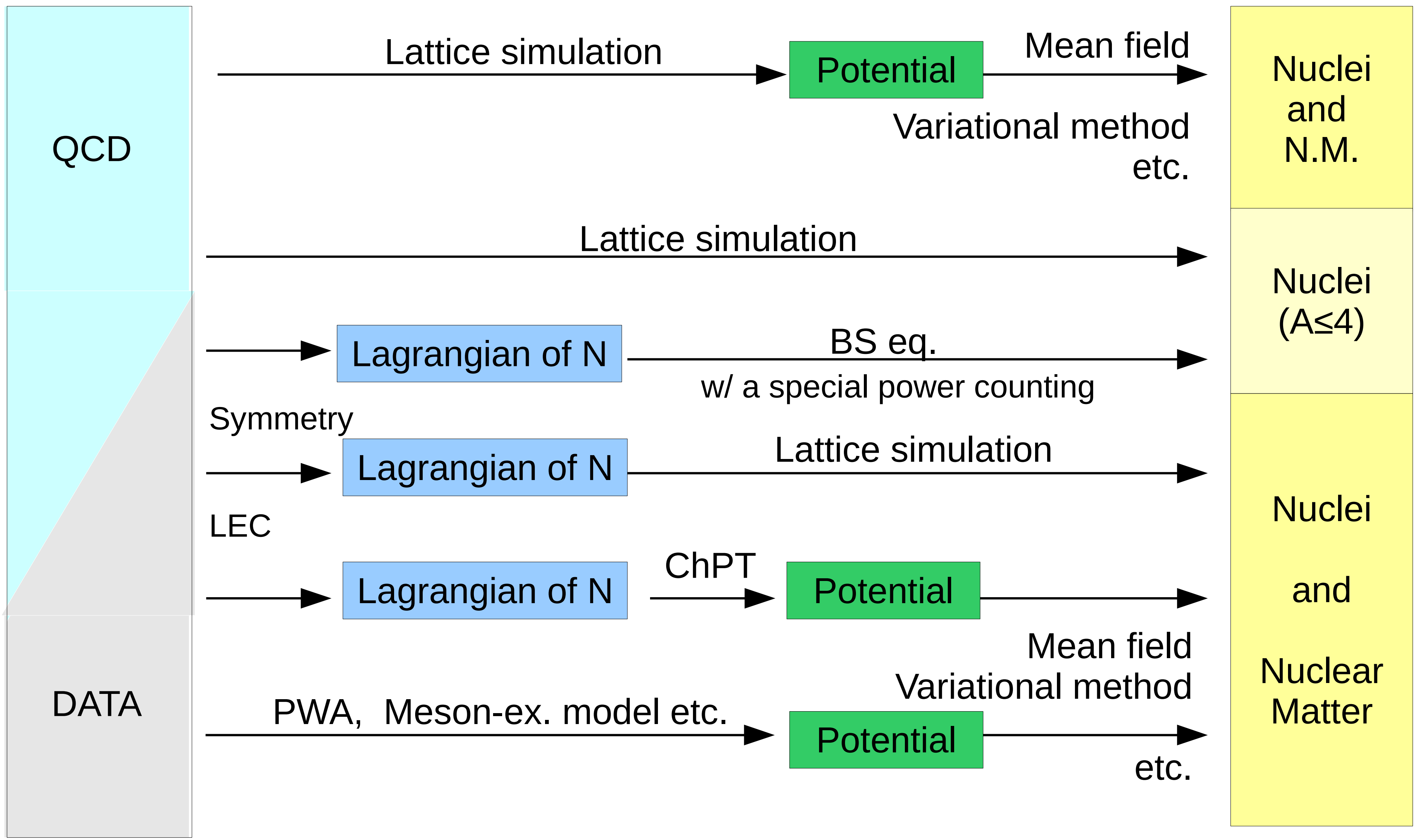


Summary by Kronfeld, arXive 1203.1204

- Mass of (ground state) hadrons are well reproduced!
- What about (hyper-)nuclei or matter from QCD?

Various approaches in nuclear phys.



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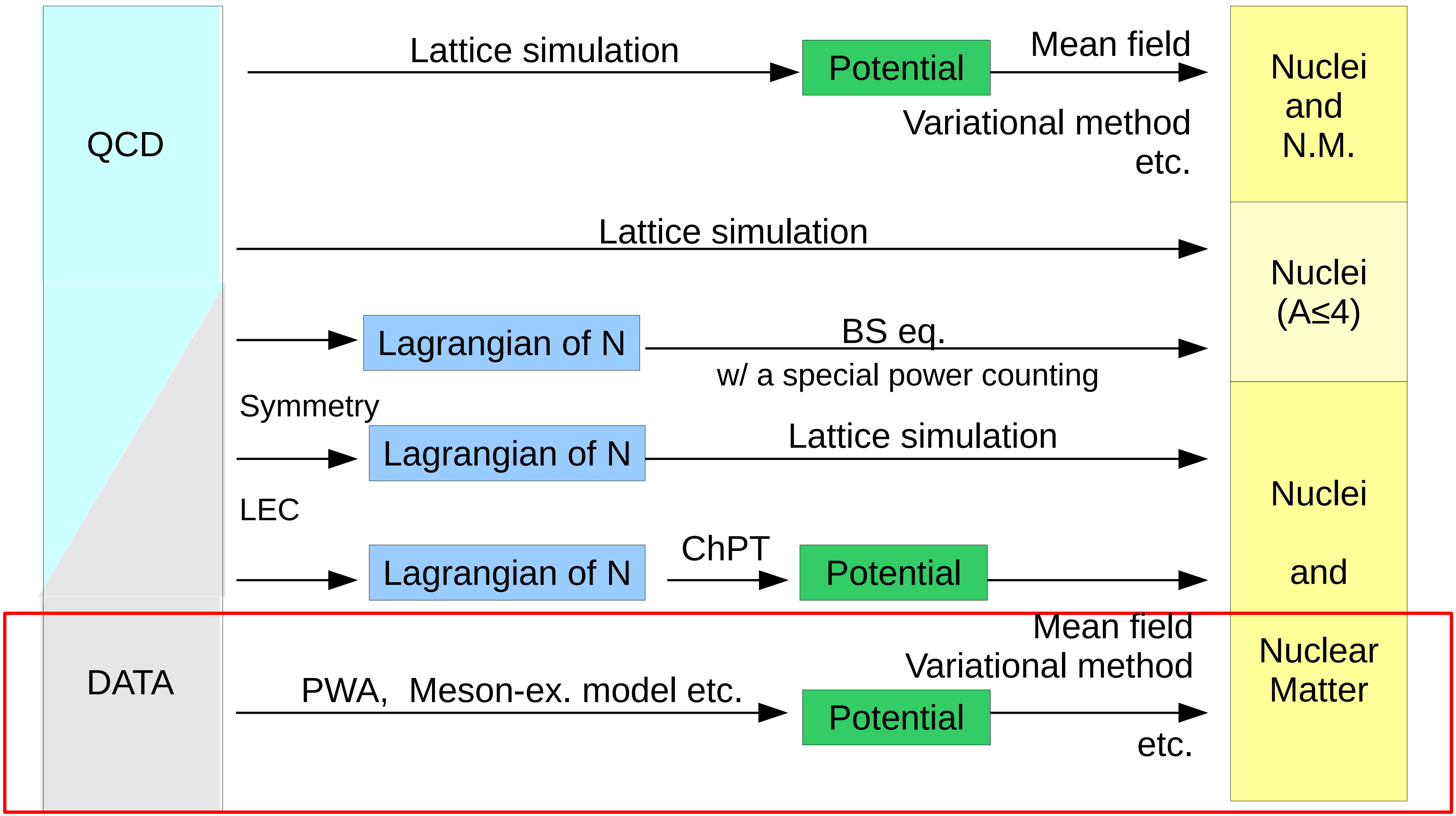
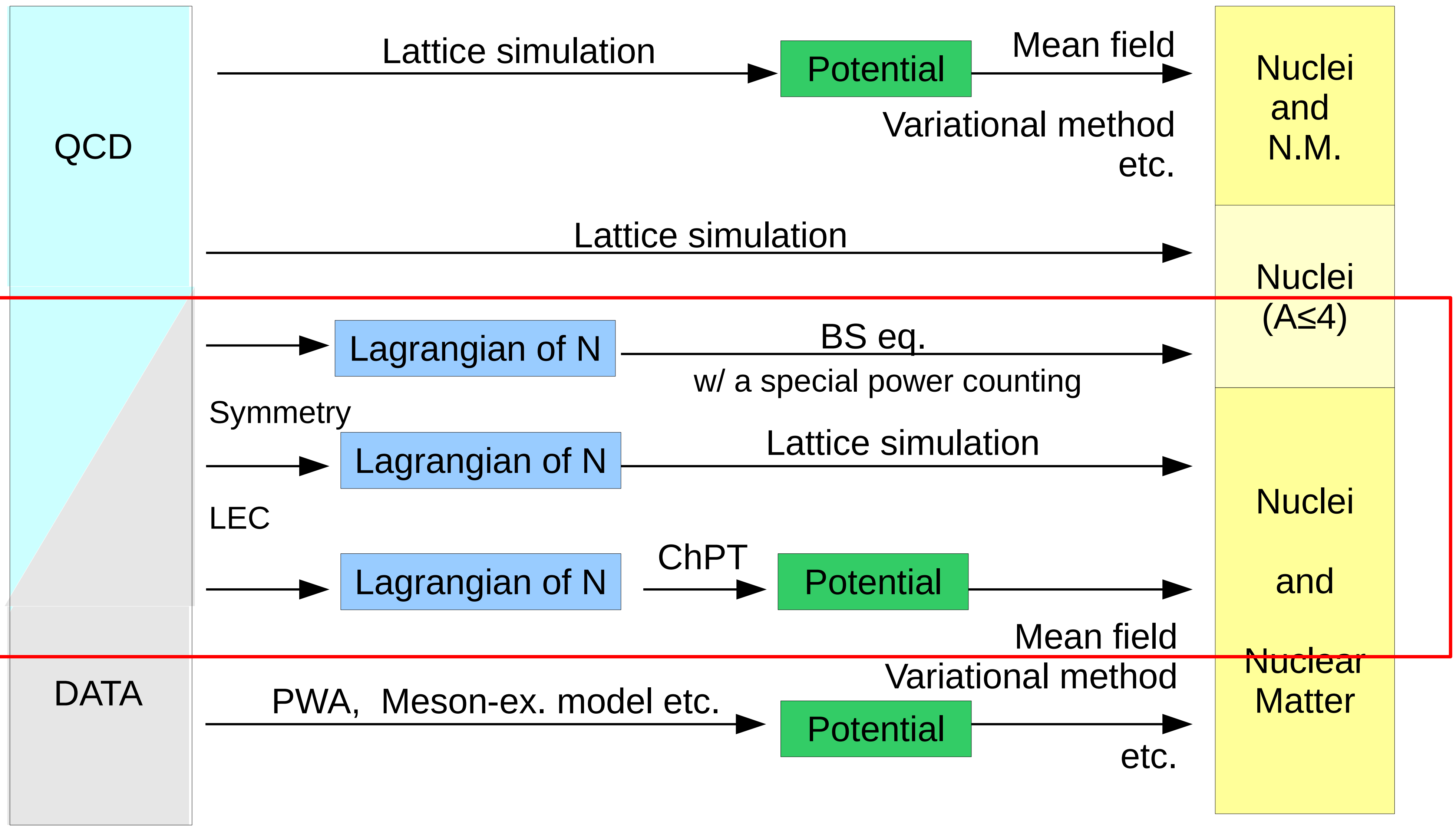


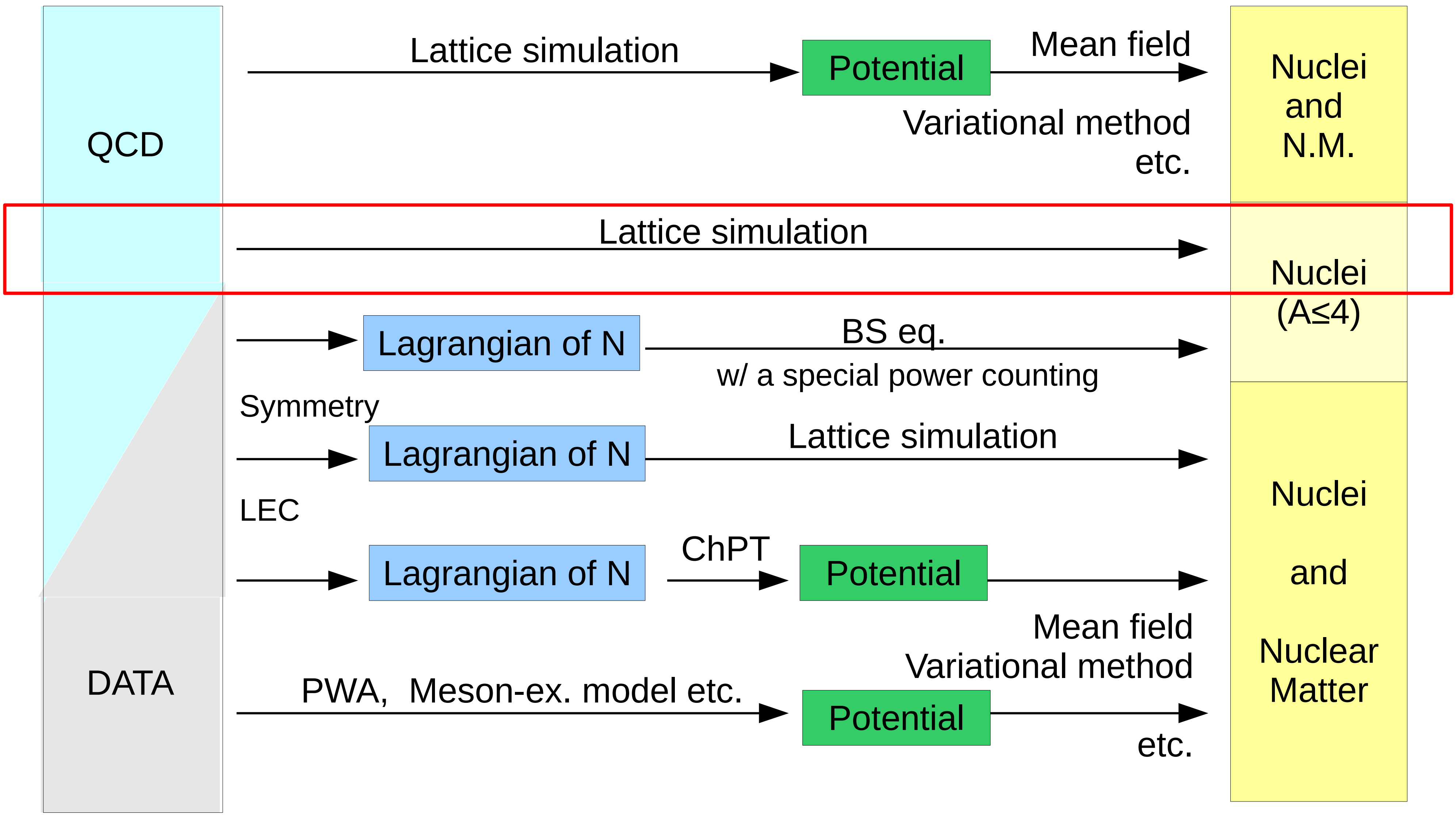
Most traditional. Many success.

Various approaches in nuclear phys.



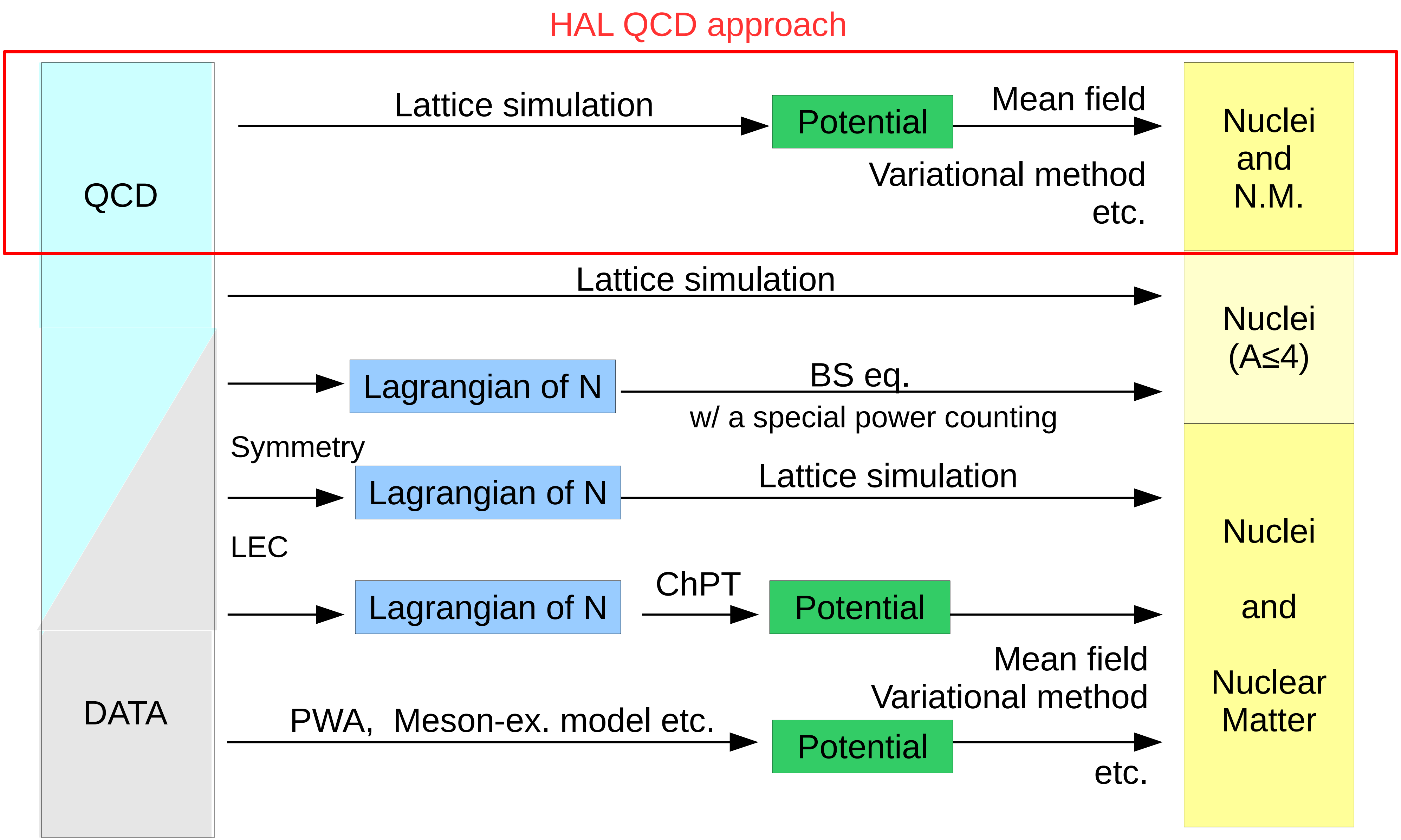
Very popular today. Let's say chiral approach.

Various approaches in nuclear phys.



Very challenging. Let's call LQCD direct approach.

Various approaches in nuclear phys.



Our approach. I focus on this one in this talk.

HAL QCD Approach

HAL QCD Approach

- Good points
  - Based on the fundamental theory QCD, hence provide information independent of experiments and models.
  - Feasible. ↔ Direct one must be very difficult for large nuclei.
  - Can utilize established nuclear theories at the 2nd stage.
  - Easy to extend to strange sector, charm sector etc.

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- Disappointing points
  1. Demand long time and huge money at the 1st stage.
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     - Un-realistically heavy u,d quark, far from chiral symmetry.
  2. Depend on method/approximation used at the 2nd stage.

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- Today, in this talk, I want to show
  - results of HALQCD approach to strange nuclear physics and want to demonstrate that our approach is promising.

Outline

1. Our approach and method
   - Introduction
   - HAL QCD method
   - Hyperon interactions from QCD

2. Application to strange nuclear physics
   - Hyperon single-particle potentials
   - Hyperon onset in high density matter

3. Summary and outlook

HAL QCD method

Multi-hadron in LQCD

- Direct : utilize energy eigenstates (eigenvalues)
  - Lüscher's finite volume method for phase-shifts
  - Infinite volume extrapolation for bound states
- HAL : utilize spatial correlation and "potential" $V(r)$ + ...

$$V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

$\psi(\vec{r},t)$: 4-point function contains NBS w.f.

  - Advantages
    - No need to separate E eigenstate. Just need to measure
    - Then, potential can be extracted.
    - Demand a minimal lattice volume. No need to extrapolate to V=∞.
    - Can output many observables.

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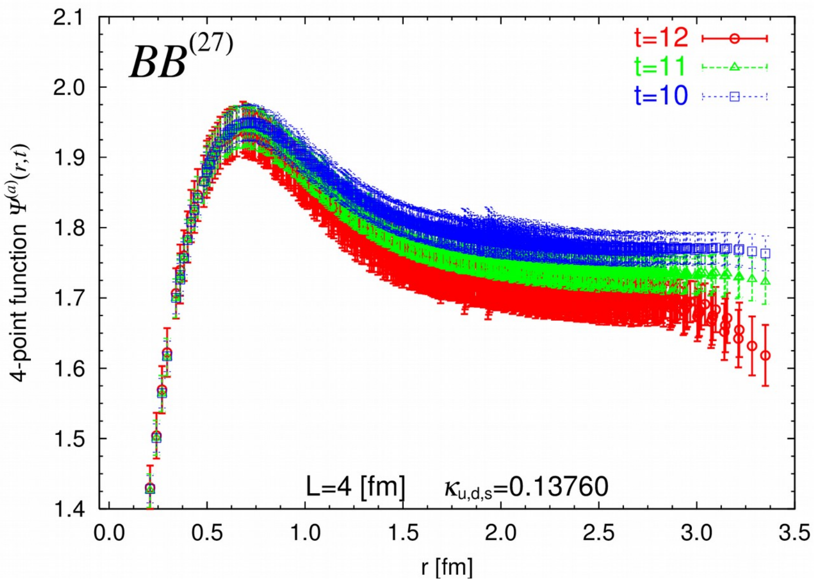

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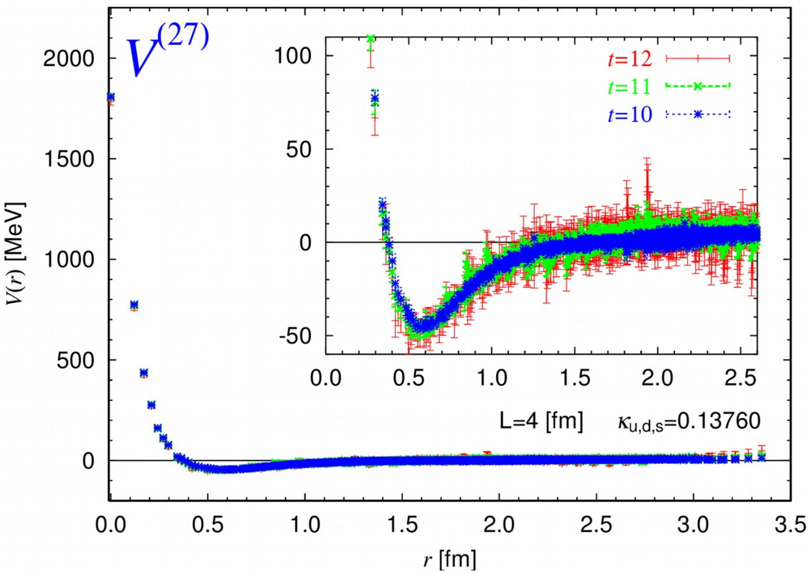

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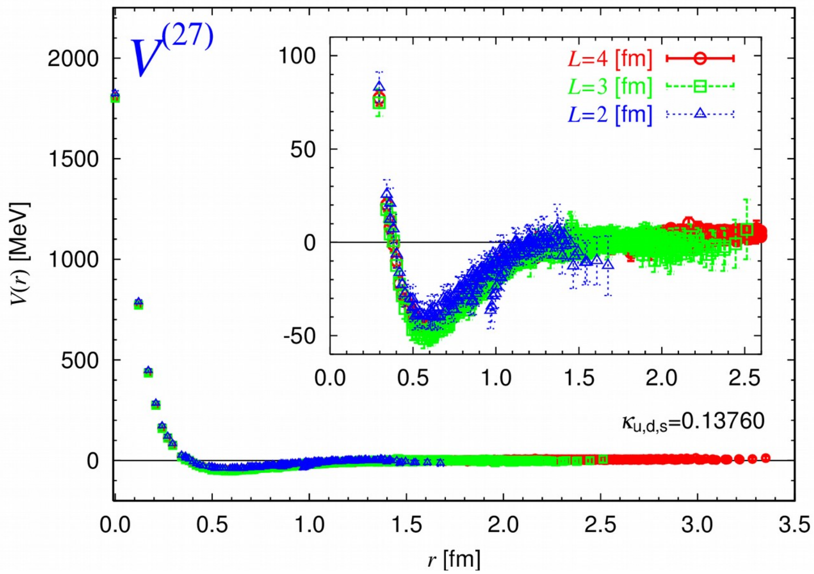

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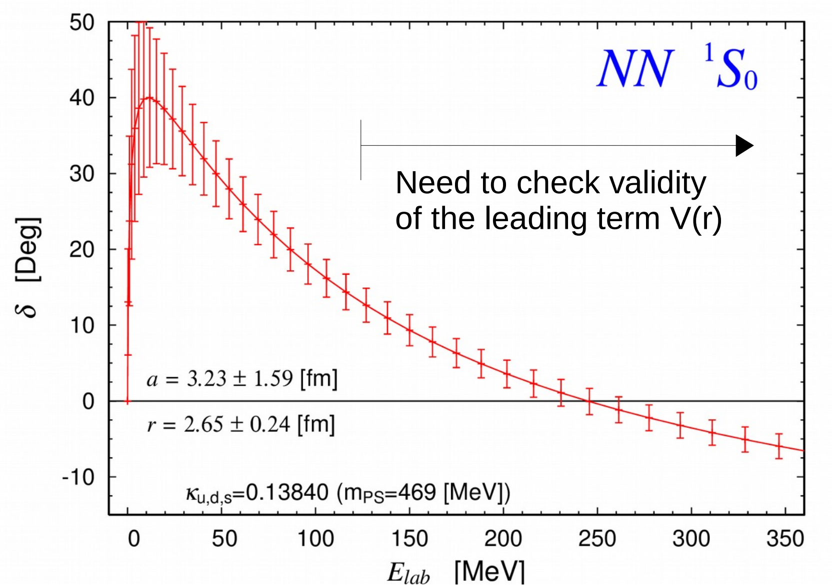

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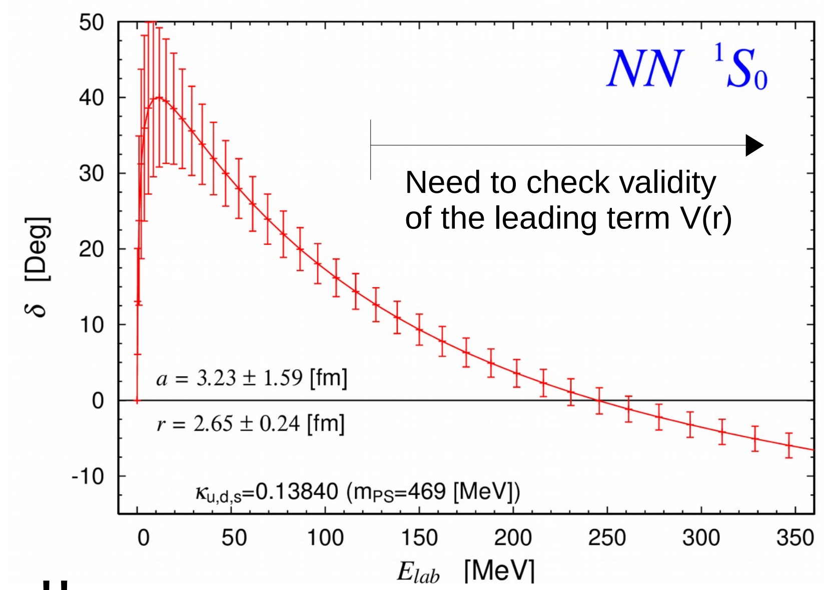


★ We can attack nuclei and matter too!!

HAL method

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theo. Phys. 123 89 (2010)
N. Ishii etal. [HAL QCD coll.] Phys. Lett. B712 , 437 (2012)

NBS wave function $\phi_{\vec{k}}(\vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x}+\vec{r},t) B_j(\vec{x},t) | B=2, \vec{k} \rangle$

Define a common potential U for all E eigenstates by a “Schrödinger” eq.

$$\left[-\frac{\nabla^2}{2\mu} \right] \phi_{\vec{k}}(\vec{r}) + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,') \phi_{\vec{k}}(\vec{r}\,') = E_{\vec{k}} \phi_{\vec{k}}(\vec{r})$$

Non-local but
energy independent
below inelastic threshold

Measure 4-point function in LQCD

$$\psi(\vec{r},t) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x}+\vec{r},t) B_j(\vec{x},t)\ J(t_0) | 0 \rangle = \sum_{\vec{k}} A_{\vec{k}} \phi_{\vec{k}}(\vec{r}) e^{-W_{\vec{k}}(t-t_0)} + \cdots$$

$$\left[2M_B - \frac{\nabla^2}{2\mu} \right] \psi(\vec{r},t) + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,') \psi(\vec{r}\,',t) = -\frac{\partial}{\partial t} \psi(\vec{r},t)$$

∇ expansion
& truncation

$$U(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,') V(\vec{r},\nabla) = \delta(\vec{r}-\vec{r}\,') [V(\vec{r}) + \cancel{\nabla + \nabla^2} \ldots]$$

Therefor, in the leading

$$V(\vec{r}) = \frac{1}{2\mu} \frac{\nabla^2 \psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t} \psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

Source and sink operator

- NBS wave function and 4-point function

$$\phi_{\vec{k}}(\vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x}+\vec{r}, \overbrace{t) B_j(\vec{x}, t}^{\text{equal}}) | B=2, \vec{k} \rangle_{\text{QCD eigenstate}}$$

$$\psi(\vec{r}, t) = \sum_{\vec{x}} \langle 0 | \underbrace{B_i(\vec{x}+\vec{r}, t) B_j(\vec{x}, t)}_{\text{sink}} \underbrace{J(t_0)}_{\text{source}} | 0 \rangle = \sum_{\vec{k}} A_{\vec{k}} \phi_{\vec{k}}(\vec{r}) e^{-W_{\vec{k}}(t-t_0)} + \cdots$$

- Point type octet baryon field operator at sink

$$p_\alpha(\underline{x}) = \epsilon_{c_1 c_2 c_3} (C\gamma_5)_{\beta_1 \beta_2} \delta_{\beta_3 \alpha} u(\xi_1) d(\xi_2) u(\xi_3) \quad \text{with} \quad \xi_i = \{c_i, \beta_i, \underline{x}\}$$

$$\Lambda_\alpha(x) = -\epsilon_{c_1 c_2 c_3} (C\gamma_5)_{\beta_1 \beta_2} \delta_{\beta_3 \alpha} \sqrt{\frac{1}{6}} \left[d(\xi_1) s(\xi_2) u(\xi_3) + s(\xi_1) u(\xi_2) d(\xi_3) - 2u(\xi_1) d(\xi_2) s(\xi_3) \right]$$

- Wall type source of two-baryon state

e.g. $\overline{BB}^{(1)} = -\sqrt{\frac{1}{8}}\, \overline{\Lambda}\,\overline{\Lambda} + \sqrt{\frac{3}{8}}\, \overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{4}{8}}\, \overline{N}\,\overline{\Xi}$ for flavor-singlet

FAQ

1. Does your potential depend on the choice of source?

2. Does your potential depend on choice of operator at sink?

3. Does your potential $U(r,r')$ or $V(r)$ depends on energy?

FAQ

1. Does your potential depend on the choice of source?

- No. Some sources may enhance excited states in 4-point func. However, it is no longer a problem in our new method.

2. Does your potential depend on choice of operator at sink?

- Yes. It can be regarded as the "scheme" to define a potential. Note that a potential itself is not physical observable. We will obtain unique result for physical observables irrespective to the choice, as long as the potential $U(r,r')$ is deduced exactly.

FAQ

3. Does your potential $U(r,r')$ or $V(r)$ depends on energy?

➔ By definition, $U(r,r')$ is non-local but energy independent. While, determination and validity of its leading term $V(r)$ depend on energy because of the truncation.

However, we know that the dependence in NN case is very small (thanks to our choice of sink operator = point) and negligible at least at $E_{lab.}$ = 0 – 90 MeV. We rely on this in our study.

If we find some dependence, we will determine the next leading term of the expansion from the dependence.

Hyperon interaction from QCD

LQCD simulation setup

- Nf = 2+1 full QCD
  - Clover fermion + Iwasaki gauge w/ stout smearing
  - Volume $96^4 \simeq (8\ \mathrm{fm})^4$
  - $1/a = 2333$ MeV, $a = 0.0845$ fm
  - $M_\pi \simeq 146$, $M_K \simeq 525$ MeV

    $M_N \simeq 956$, $M_\Lambda \simeq 1121$, $M_\Sigma \simeq 1201$, $M_\Xi \simeq 1328$ MeV
  - Collaboration in HPCI Strategic Program Field 5 Project 1

K-configuration

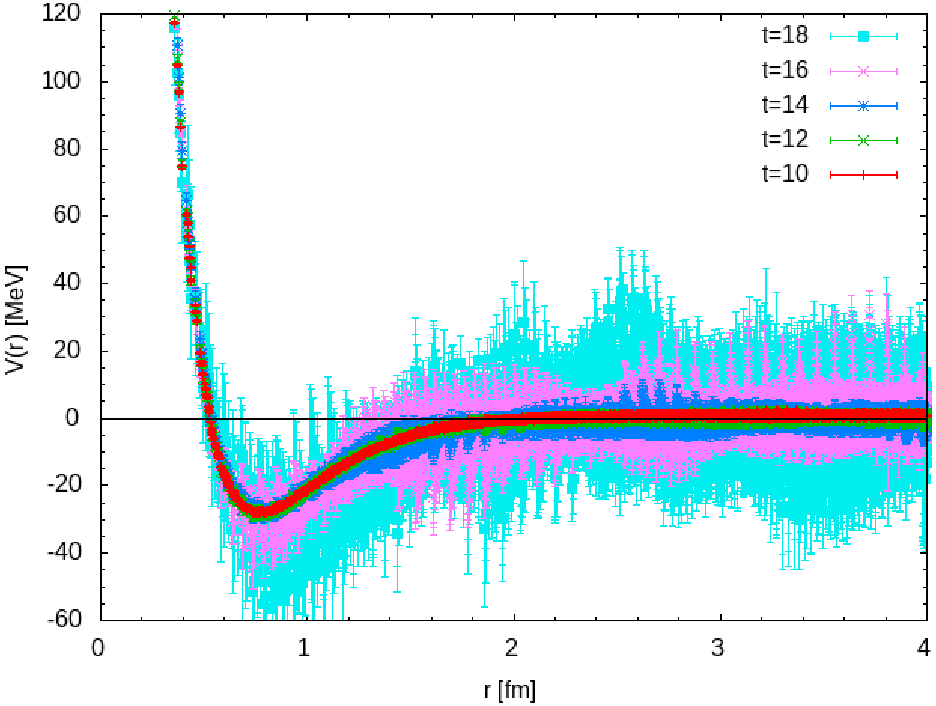
almost physical point

- Measurement
  - 4pt correlators: 52 channels in 2-octet-baryon (+ others)
  - Wall source w/ Coulomb gauge fixing
  - Dirichlet temporal BC to avoid the wrap around artifact
  - #stat = 414 confs $\times$ 4 rot $\times$ 28 src.

Not final. We are still increasing #stat.

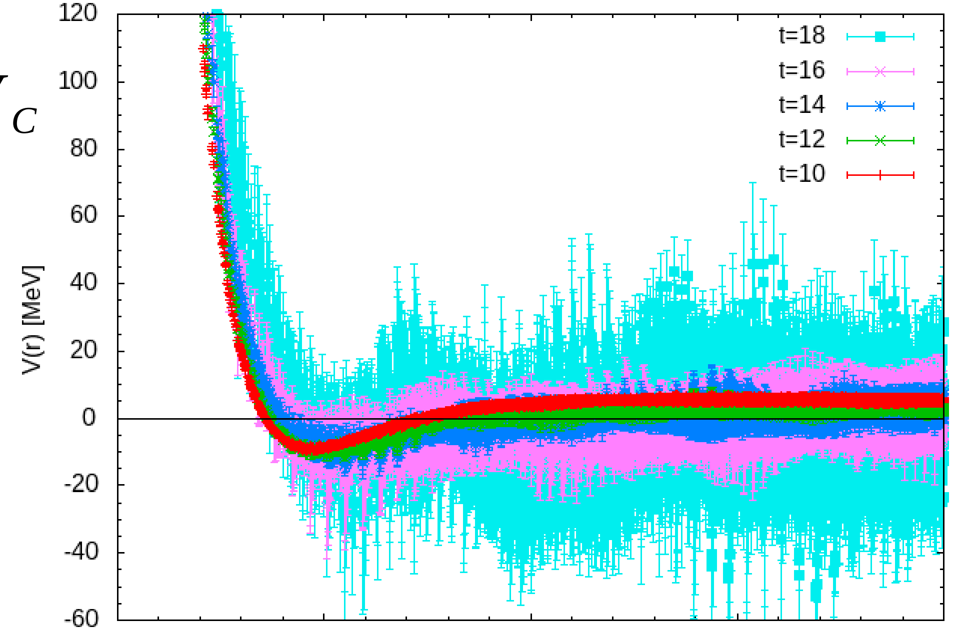
ΞΞ S=−4

$^1S_0$

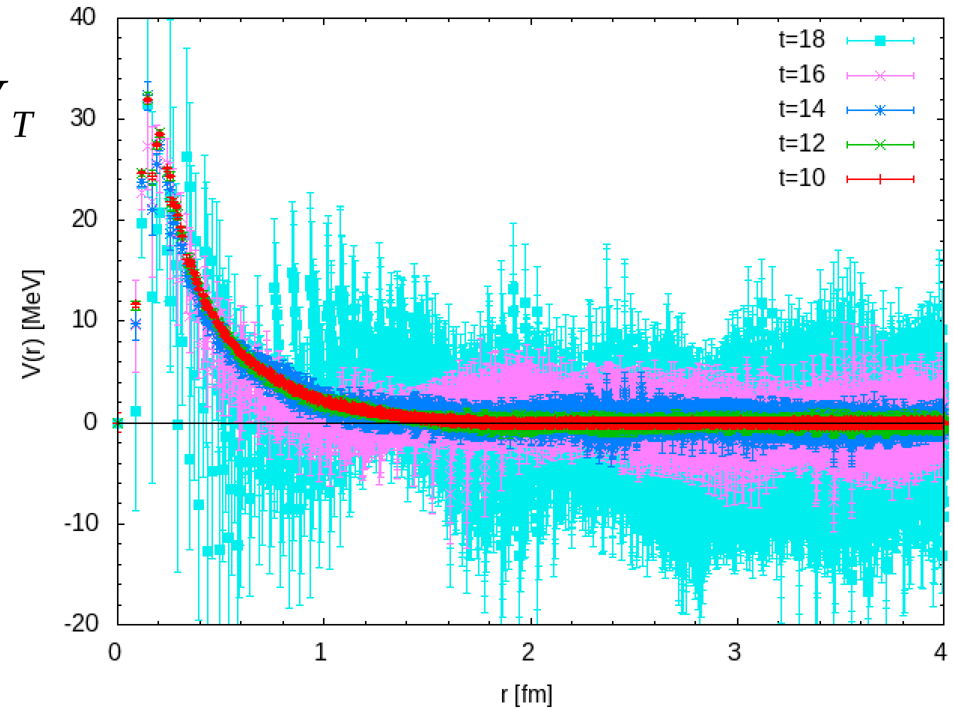


$^3SD_1$

V_C



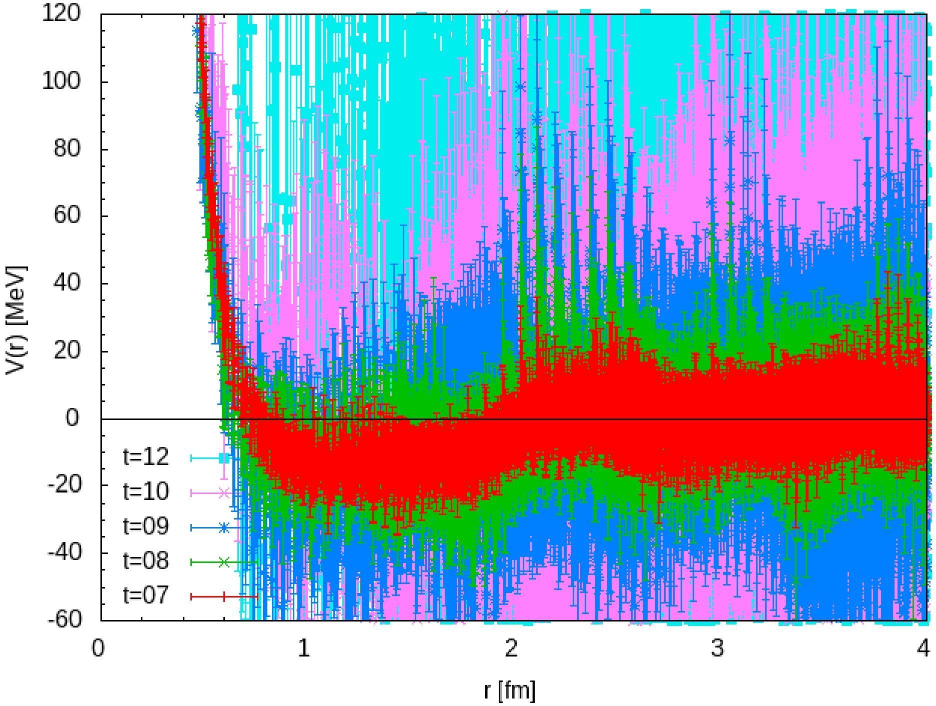
V_T



- Most clean case in B8B8
- $t - t_0 = 10$ - 18
- 27 & 10 in flavor SU(3) limit

NN S=0

$^1S_0$



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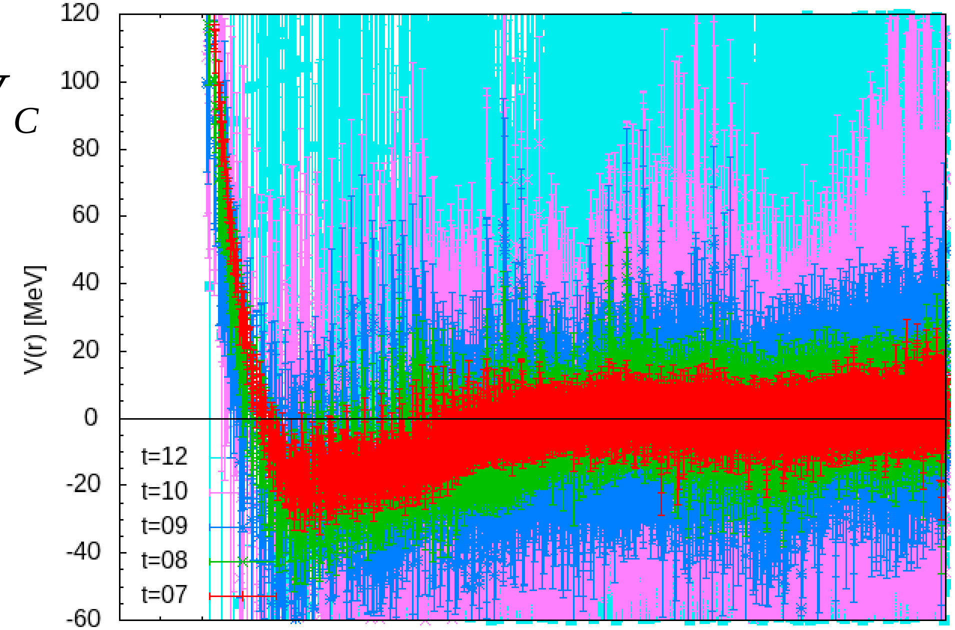
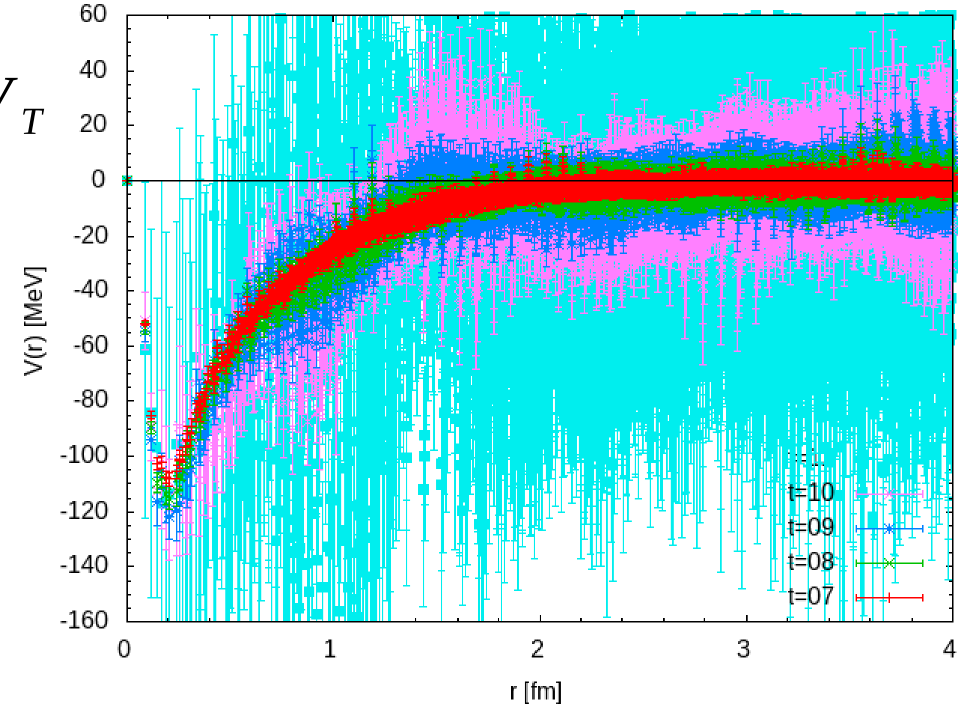
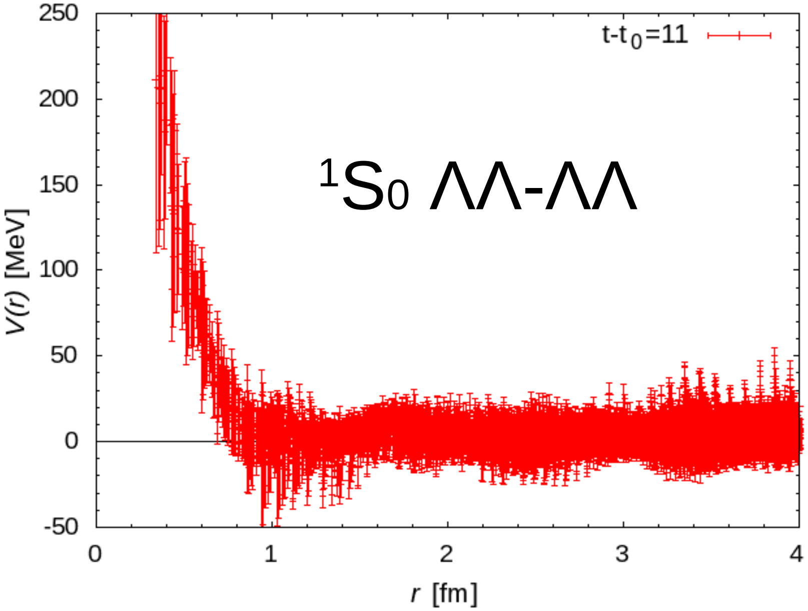


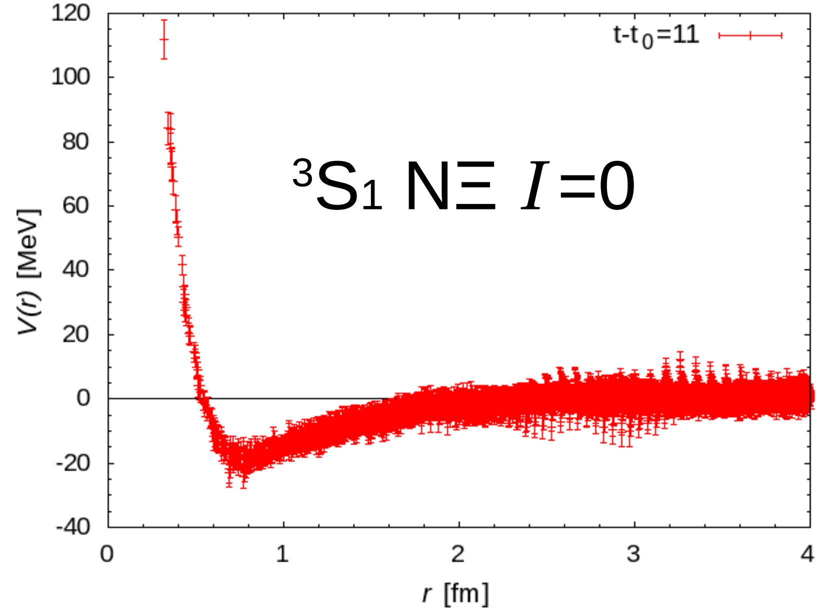
V_T



- Most noizy case in B8B8
- $t\!-\!t_0 = 7$ - 12
- 27 & 10* in flavor SU(3) limit

YY,YN S=−2



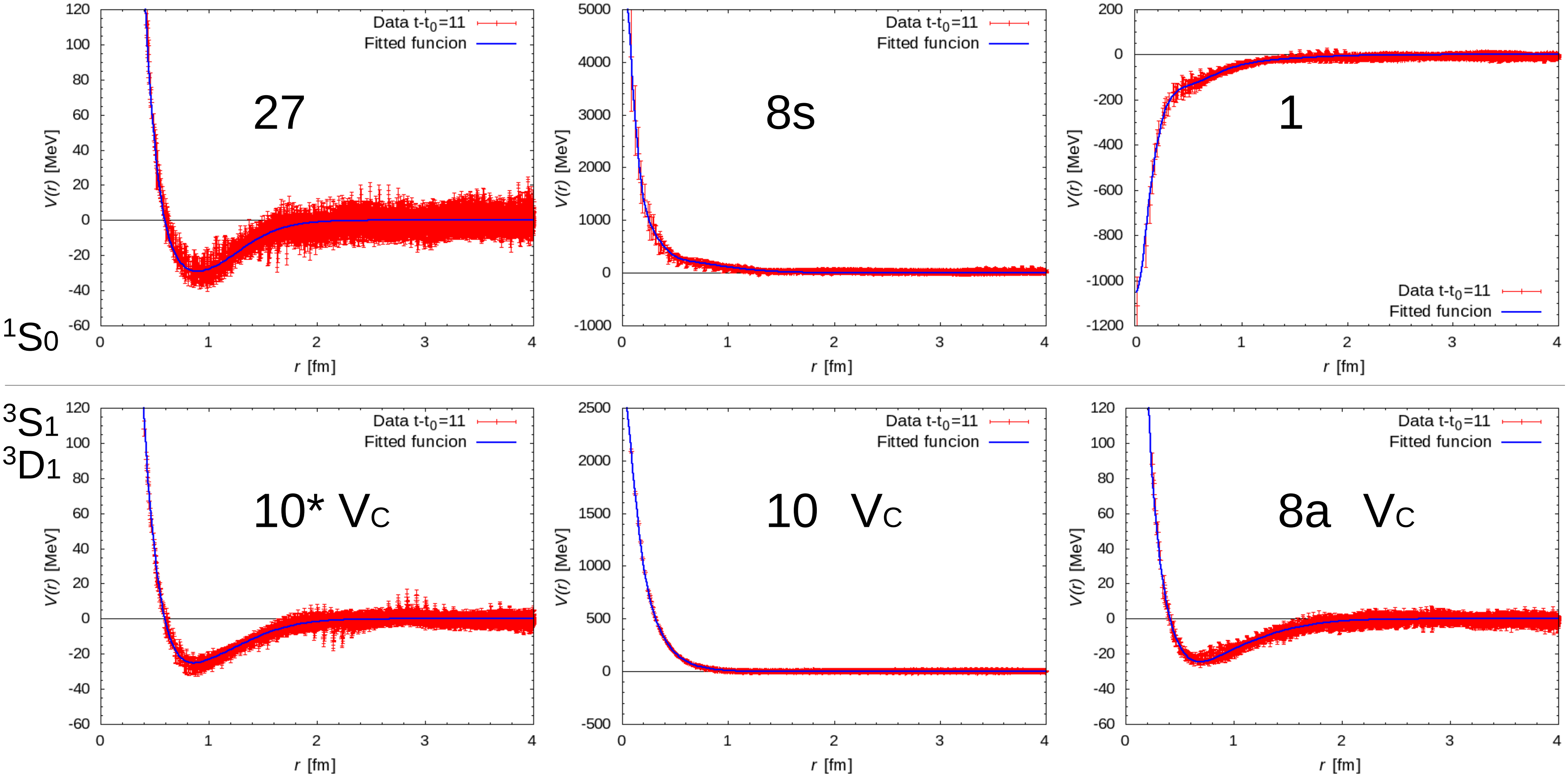


etc. for example

- There are many particle-base potentials. #≈100 in S-wave.
- For application, we need to parameterize potential data.
- It is tough to parameterize all needed potential data.
- So, today, for the moment, I use potential data rotated into the irreducible-representation base.

$$8 \times 8 = \underbrace{27 + 8s + 1}_{{}^1S_0} + \underbrace{10^* + 10 + 8a}_{{}^3S_1\,,\ {}^3D_1}$$

Irre.-rep. base diagonal potentials

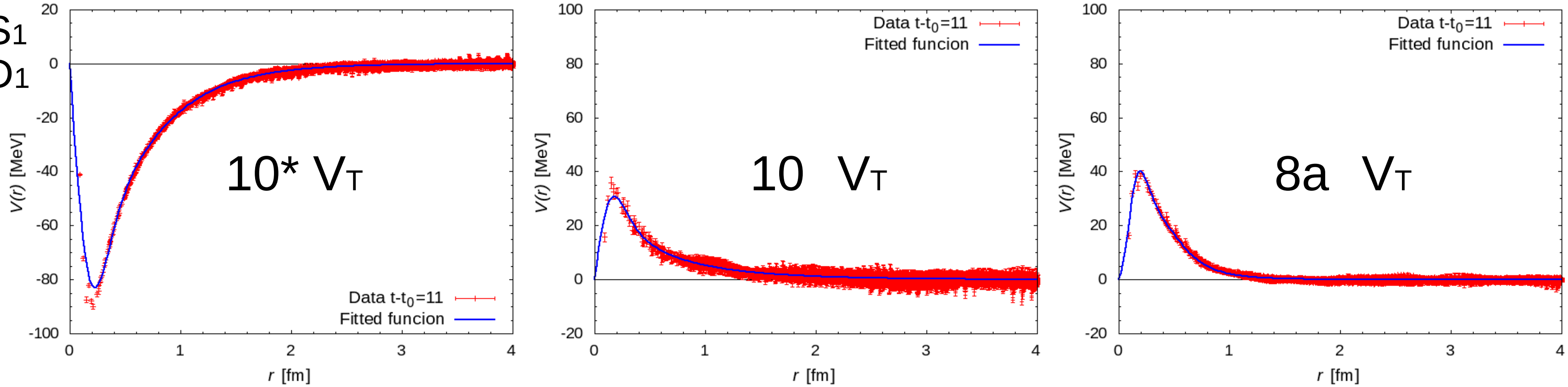


- Analitic function fitted to data

$$V(r) = a_1\, e^{-a_2 r^2} + a_3\, e^{-a_4 r^2} + a_5 \left[\left(1 - e^{-a_6 r^2}\right) \frac{e^{-a_7 r}}{r} \right]^2$$

Irre.-rep. base diagonal potentials

$^3S_1$
$^3D_1$



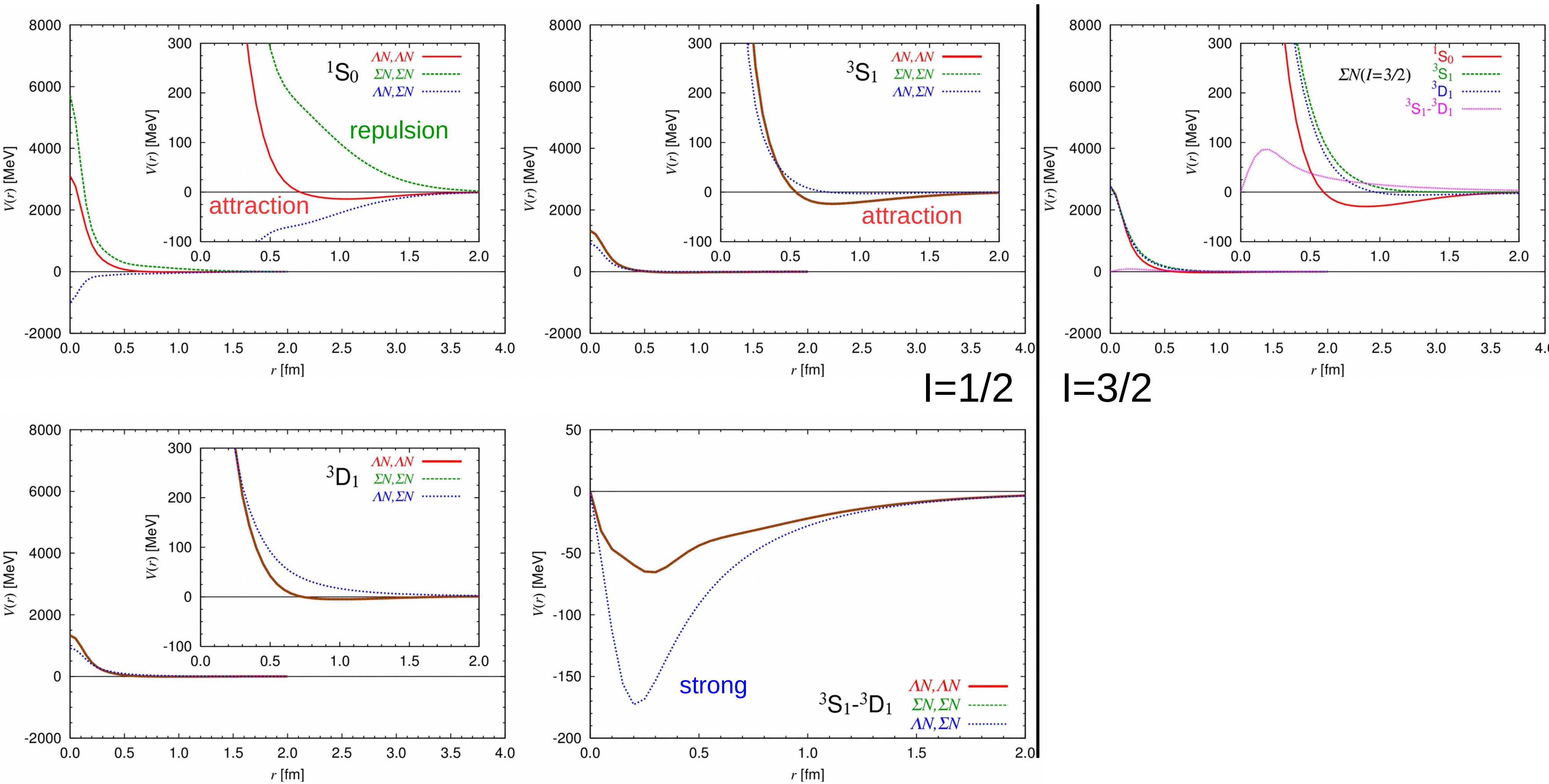
- Analitic function fitted to data

$$V(r) = a_1 \left(1-e^{-a_2 r^2}\right)^2 \left(1 + \frac{3}{a_3 r} + \frac{3}{(a_3 r)^2}\right) \frac{e^{-a_3 r}}{r} + a_4 \left(1-e^{-a_5 r^2}\right)^2 \left(1 + \frac{3}{a_6 r} + \frac{3}{(a_6 r)^2}\right) \frac{e^{-a_6 r}}{r}$$

- Since SU(3)$_F$ is broken at the physical point (K-conf.), there are irre.-rep. base off-diagonal potentials.
- But, I omit them and constract V_{YN}, V_{YY} with these irre.-rep. diagonal potentials and the C.G. coefficient.

LQCD ΛN-ΣN

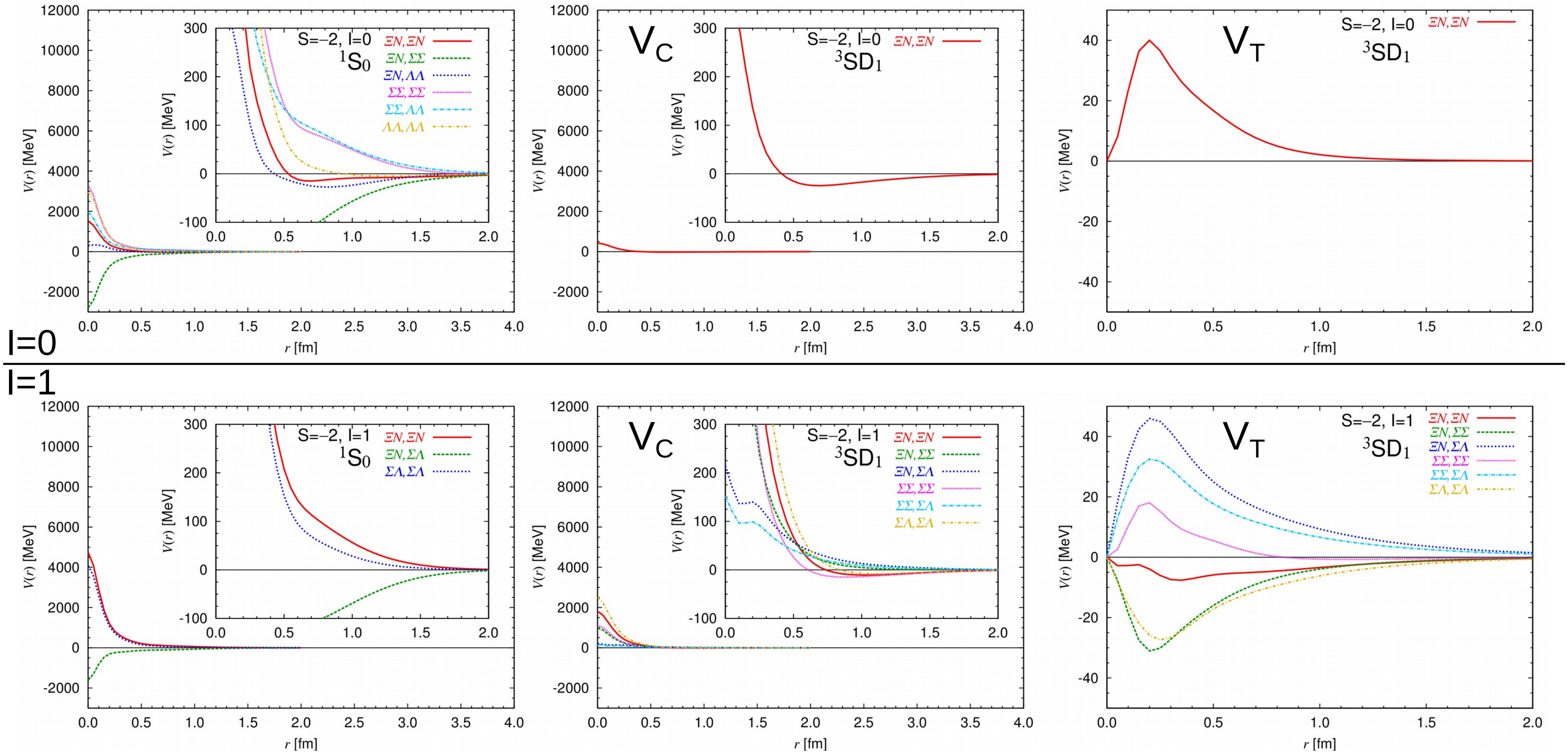
From K-conf. but rotated from the irr.-rep. base diagonal potentials.



- In I=1/2, $^1S_0$ channel, ΛN has an attraction, while ΣN is repulsive.
- In I=1/2, $^3S_1$ channel, both ΛN and ΣN have an attraction. ↔ No attraction in Nijmegen
- In I=1/2, strong tensor coupling in flavor off-diagonal.

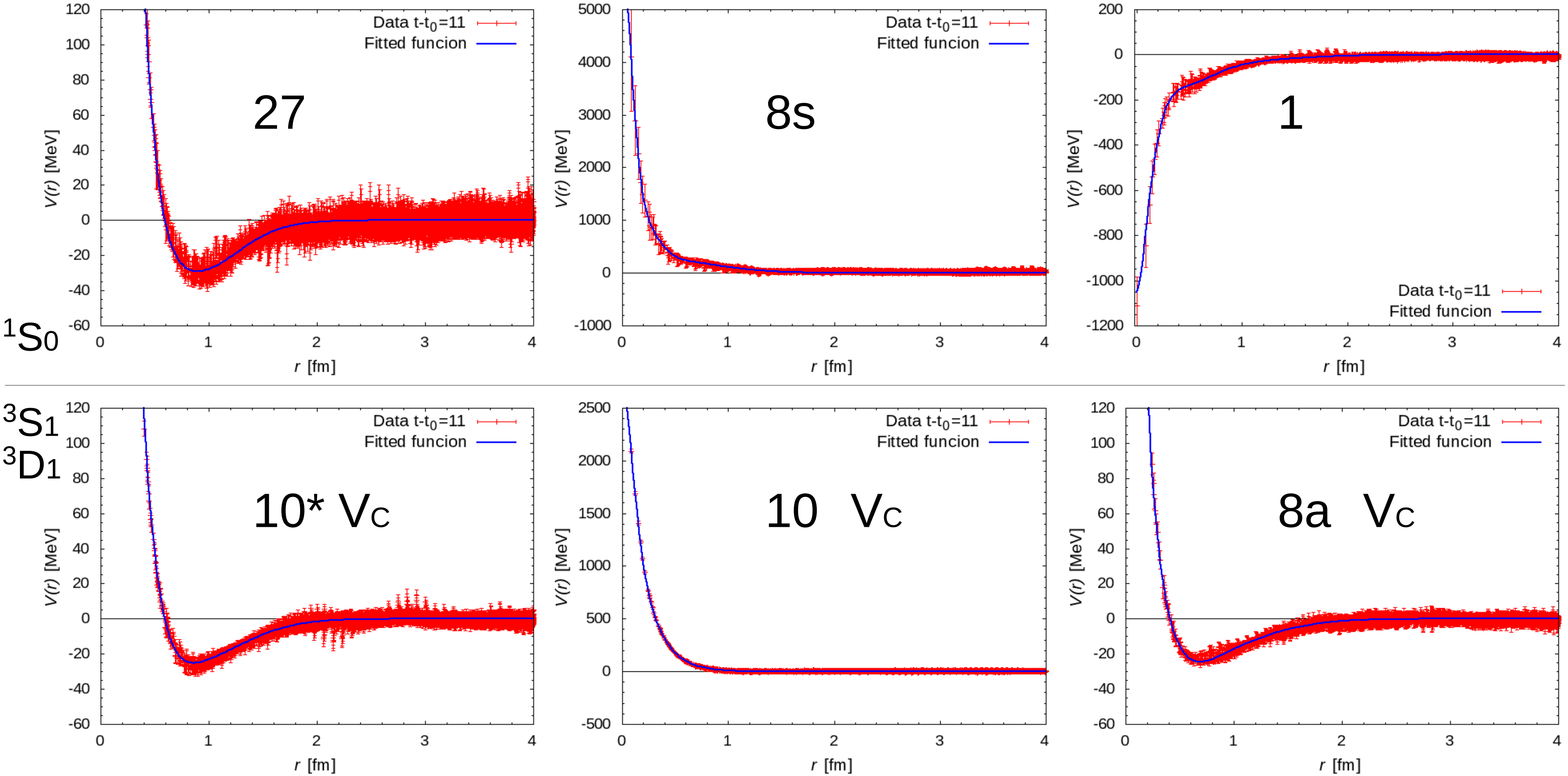
LQCD ΞN-YY

From K-conf. but rotated from the irr.-rep. base diagonal potentials.



- Many experimentally unknown coupled-channel potentials.
- One can see predictive power of the HALQCD method.

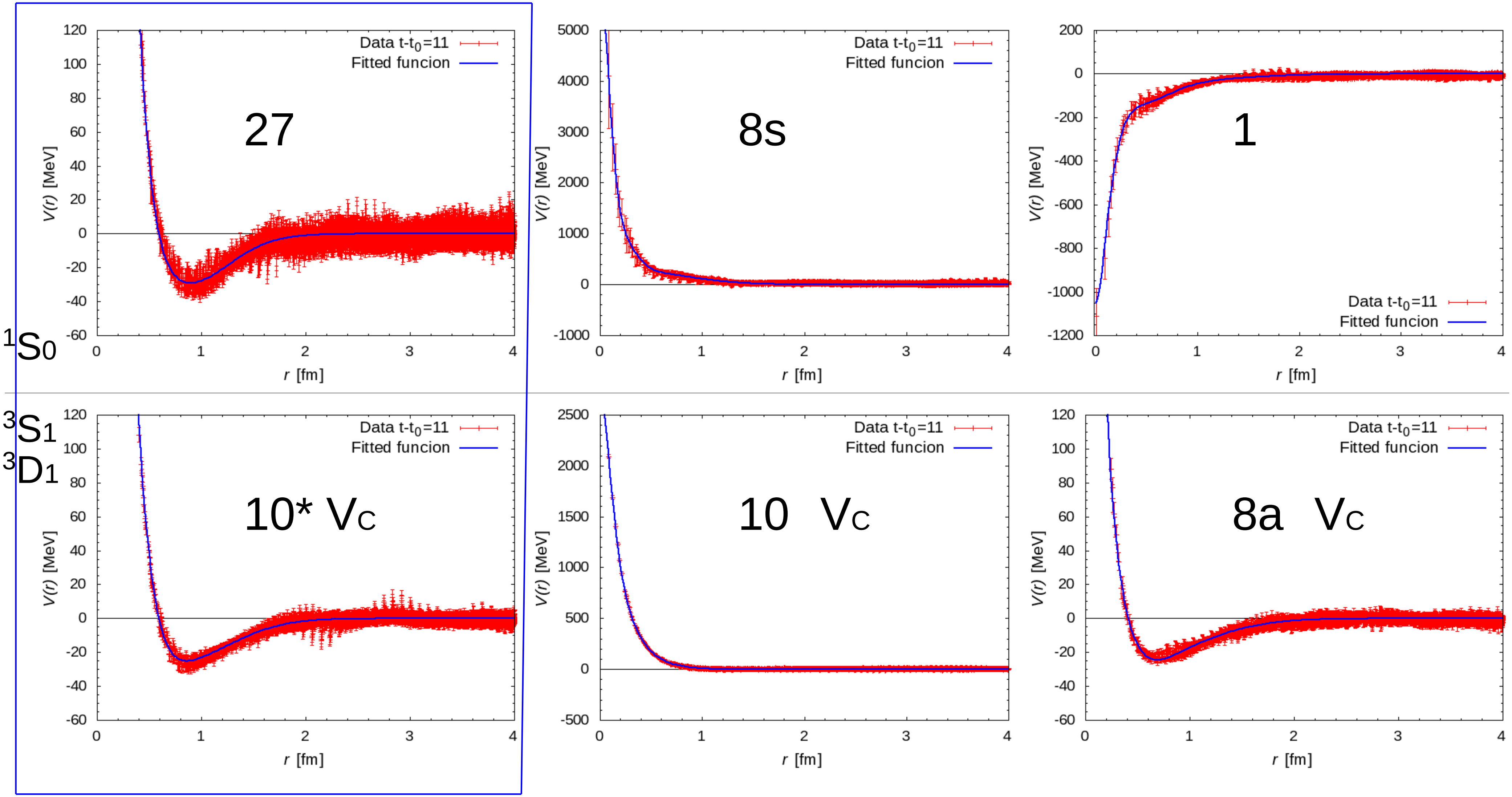
Irre.-rep. base diagonal potentials



- Analitic function fitted to data

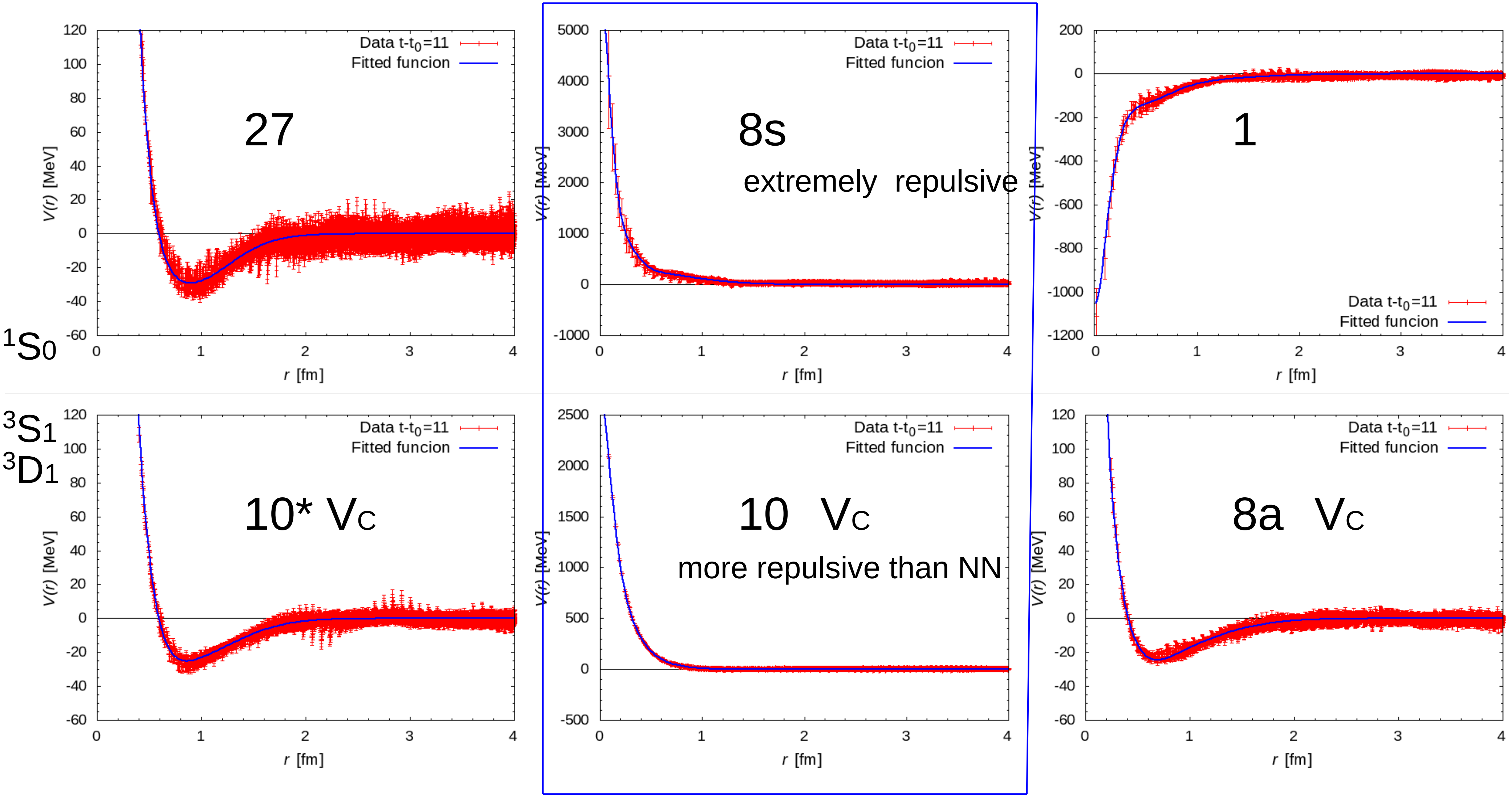
$$V(r) = a_1\, e^{-a_2 r^2} + a_3\, e^{-a_4 r^2} + a_5 \left[\left(1 - e^{-a_6 r^2}\right) \frac{e^{-a_7 r}}{r} \right]^2$$

Irre.-rep. base diagonal potentials



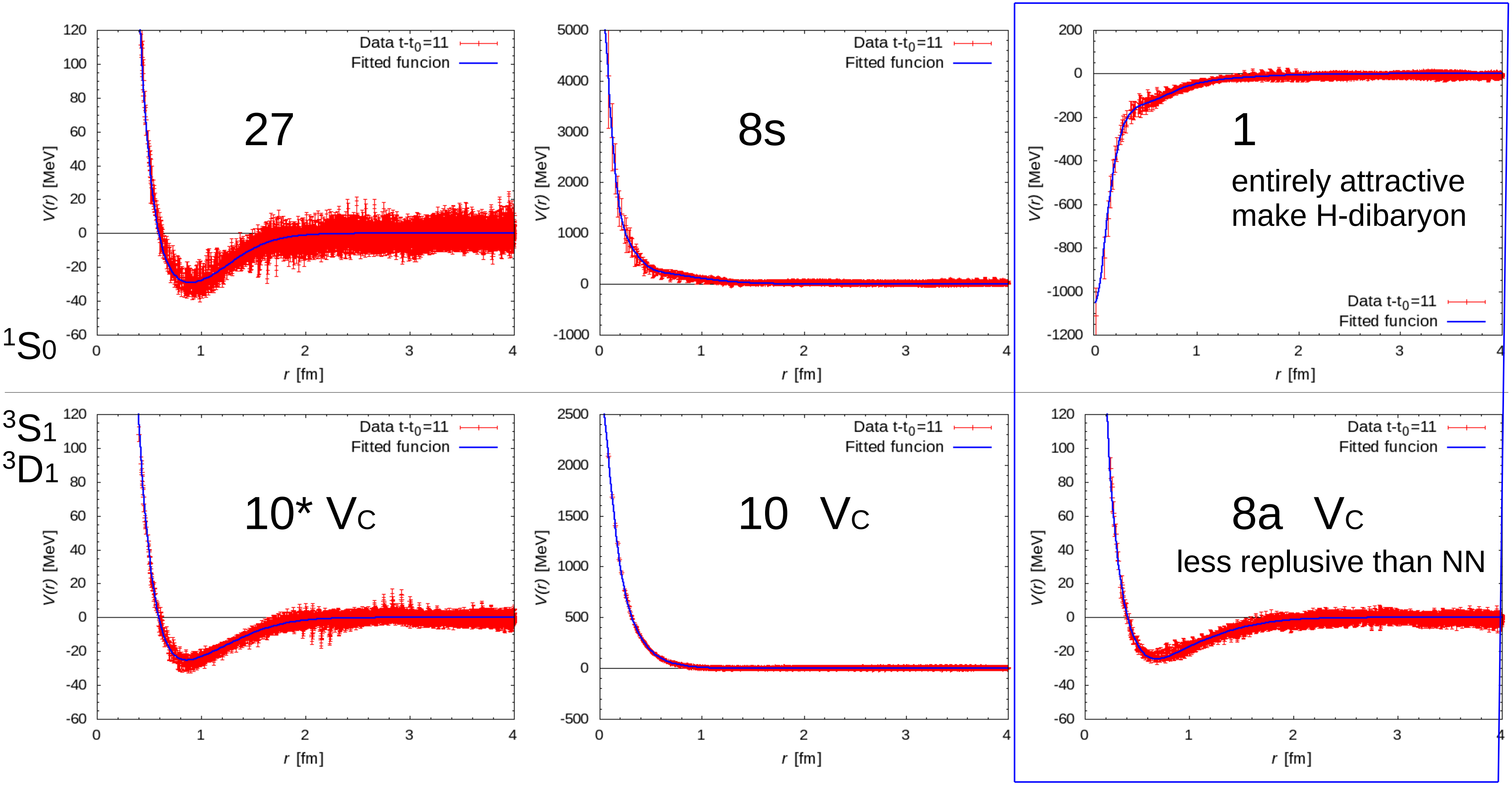
- Nothing but a nuclear force in the SU(3)$_F$ limit.
- They are qualitatively reasonable. repulive core, attractive pocket and stron tensor

Irre.-rep. base diagonal potentials



- Can be understood by the quark pauli effect.

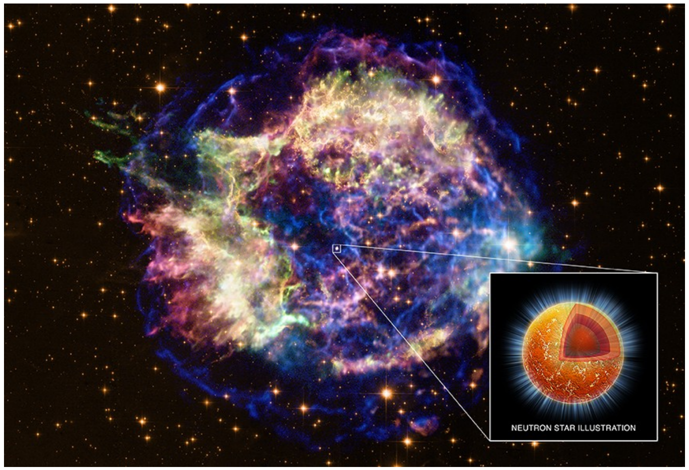
Irre.-rep. base diagonal potentials

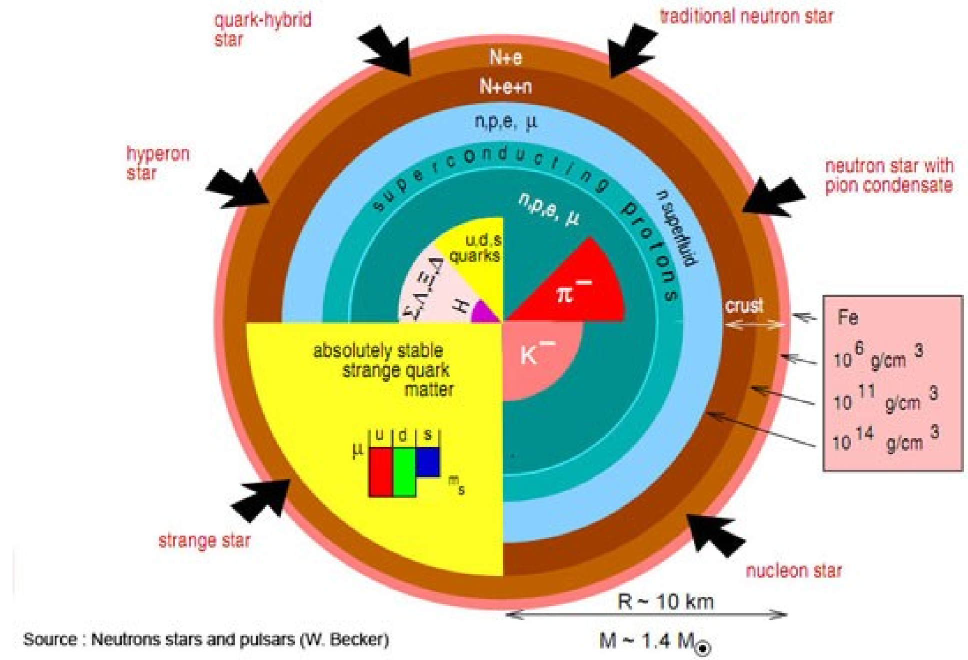


- Can be understood by the quark pauli effect.

Part 2.
Application to strange nuclear physics

Introduction





☆ Hyperon is a serious subject in physics of NS.

- Does hyperon appear inside neutron star core?
- How EoS of NS mater can be so stiff with hyperon

  cf. PSR J1614-2230 1.97±0.04 $M_\odot$

- Tough problem due to ambiguity of hyperon forces
  - comes form difficulty of hyperon scattering experiment.

In my talk I focus on hadron phase

Introduction

- However, nowadays, we can study or predict hadron-hadron interactions from QCD.
  - measure h-h NBS w.f. in lattice QCD simulation.
  - define & extract interaction “potential” from the w.f.

HALQCD
applapch

Introduction

- However, nowadays, we can study or predict hadron-hadron interactions from QCD.
  - measure h-h NBS w.f. in lattice QCD simulation.
  - define & extract interaction "potential" from the w.f.

HALQCD applapch

- Today, we study hyperons in nuclear medium by basing on YN YY interactions predicted from QCD.
  - We calculate hyperon single-particle potential $U_Y(k;\rho)$
  - defined by $$e_Y(k;\rho) = \frac{k^2}{2M_Y} + U_Y(k;\rho)$$ $e_Y(k;\rho)$: sepectrum in medium
  - U_Y is crucial for hyperon chemical potential.

Introduction

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HALQCD
applapch

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  - U_Y is crucial for hyperon chemical potential.

- Hypernuclear experiment suggest that $@\rho = 0.17\,[\mathrm{fm}^{-3}]$, $x = 0.5$

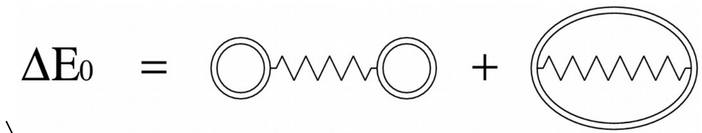
$$U^{\mathrm{Exp}}_{\Lambda}(0) \simeq -30, \quad U^{\mathrm{Exp}}_{\Xi}(0) \simeq -10, \quad U^{\mathrm{Exp}}_{\Sigma}(0) \geq +20 \quad [\mathrm{MeV}]$$

attraction attraction small repulsionl

Brueckner-Hartree-Fock LOBT

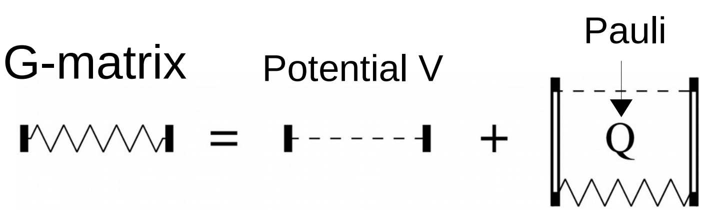
M.I. Haftel and F. Tabakin, Nucl. Phys. A158(1970) 1-42

- Ground state energy in BHF framework



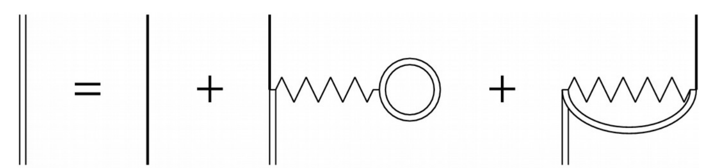
$$E_0 = \gamma \sum_{k}^{k_F} \frac{k^2}{2M} + \frac{1}{2} \sum_{i}^{N_{ch}} \sum_{k,k'}^{k_F} \mathrm{Re} \langle\, G_i\big(e(k)+e(k')\big) \,\rangle_A$$

- G-matrix



$$\langle k_1 k_2 | G(\omega) | k_3 k_4 \rangle = \langle k_1 k_2 | V | k_3 k_4 \rangle + \sum_{k_5, k_6} \frac{\langle k_1 k_2 | V | k_5 k_6 \rangle Q(k_5, k_6) \langle k_5 k_6 | G(\omega) | k_3 k_4 \rangle}{\omega - e(k_5) - e(k_6)}$$

- Single particle spectrum & potential



$$e(k) = \frac{k^2}{2M} + U(k)$$

$$U(k) = \sum_{i} \sum_{k' \le k_F} \mathrm{Re} \langle k\, k' | G_i\big(e(k)+e(k')\big) | k\, k' \rangle_A$$

- p.w. decomposition & truncation ${}^{2S+1}L_J = {}^1S_0\,,\, {}^3S_1\,,\, {}^3D_1\,,\, {}^1P_1\,,\, {}^3P_J \cdots$
- Continuous choice w/ effective mass approx. Angle averaged Q-operator

Brueckner-Hartree-Fock LOBT

- Hyperon single-particle potential

M. Baldo, G.F. Burgio, H.-J. Schulze, Phys. Rev. C58, 3688 (1998)

$$U_Y(k) = \sum_{N=n,p} \sum_{SLJ} \sum_{k' \le k_F} \langle k\, k' | G^{SLJ}_{(YN)(YN)}\left(e_Y(k)+e_N(k')\right) | k\, k' \rangle$$

$$^{2S+1}L_J = {}^1S_0,\ {}^3S_1,\ {}^3D_1,\ \Big|\ {}^1P_1,\ {}^3P_J\ \cdots$$

in our study

limitation

- YN G-matrix using $V^{\mathrm{LQCD}}_{S=-1}$, $M^{\mathrm{Phys}}_{N,Y}$, $U^{\mathrm{AV18,BHF}}_{n,p}$ and, U^{LQCD}_{Y}

$$Q=0 \quad \begin{pmatrix} G^{SLJ}_{(\Lambda n)(\Lambda n)} & G_{(\Lambda n)(\Sigma^0 n)} & G_{(\Lambda n)(\Sigma^- p)} \\ G_{(\Sigma^0 n)(\Lambda n)} & G_{(\Sigma^0 n)(\Sigma^0 n)} & G_{(\Sigma^0 n)(\Sigma^- p)} \\ G_{(\Sigma^- p)(\Lambda n)} & G_{(\Sigma^- p)(\Sigma^0 n)} & G_{(\Sigma^- p)(\Sigma^- p)} \end{pmatrix} \qquad Q=+1 \quad \begin{pmatrix} G^{SLJ}_{(\Lambda p)(\Lambda p)} & G_{(\Lambda p)(\Sigma^0 p)} & G_{(\Lambda p)(\Sigma^+ n)} \\ G_{(\Sigma^0 p)(\Lambda p)} & G_{(\Sigma^0 p)(\Sigma^0 p)} & G_{(\Sigma^0 p)(\Sigma^+ n)} \\ G_{(\Sigma^+ n)(\Lambda p)} & G_{(\Sigma^+ n)(\Sigma^0 p)} & G_{(\Sigma^+ n)(\Sigma^+ n)} \end{pmatrix}$$

$$Q=-1 \quad G^{SLJ}_{(\Sigma^- n)(\Sigma^- n)} \qquad Q=+2 \quad G^{SLJ}_{(\Sigma^+ p)(\Sigma^+ p)}$$

Brueckner-Hartree-Fock

- Hyperon single-particle potential

$$U_{\Xi}(k) = \sum_{N=n,p} \sum_{SLJ} \sum_{k' \leq k_F} \langle k\, k' | \, G^{SLJ}_{(\Xi N)(\Xi N)}\big(e_{\Xi}(k) + e_N(k')\big) \, | k\, k' \rangle$$

- ΞN G-matrix using $V^{\mathrm{LQCD}}_{S=-2}$, $M^{\mathrm{Phys}}_{N,Y}$, $U^{\mathrm{AV18}}_{n,p}$, $U^{\mathrm{LQCD}}_{\Lambda,\Sigma}$ and, U^{LQCD}_{Ξ}

Flavor symmetric $^1S_0$ sectors

Q=0

$$\begin{pmatrix}
G^{SLJ}_{(\Xi^0 n)(\Xi^0 n)} & G_{(\Xi^0 n)(\Xi^- p)} & G_{(\Xi^0 n)(\Sigma^+\Sigma^-)} & G_{(\Xi^0 n)(\Sigma^0\Sigma^0)} & G_{(\Xi^0 n)(\Sigma^0\Lambda)} & G_{(\Xi^0 n)(\Lambda\Lambda)} \\
G_{(\Xi^- p)(\Xi^0 n)} & G_{(\Xi^- p)(\Xi^- p)} & G_{(\Xi^- p)(\Sigma^+\Sigma^-)} & G_{(\Xi^- p)(\Sigma^0\Sigma^0)} & G_{(\Xi^- p)(\Sigma^0\Lambda)} & G_{(\Xi^- p)(\Lambda\Lambda)} \\
G_{(\Sigma^+\Sigma^-)(\Xi^0 n)} & G_{(\Sigma^+\Sigma^-)(\Xi^- p)} & G_{(\Sigma^+\Sigma^-)(\Sigma^+\Sigma^-)} & G_{(\Sigma^+\Sigma^-)(\Sigma^0\Sigma^0)} & G_{(\Sigma^+\Sigma^-)(\Sigma^0\Lambda)} & G_{(\Sigma^+\Sigma^-)(\Lambda\Lambda)} \\
G_{(\Sigma^0\Sigma^0)(\Xi^0 n)} & G_{(\Sigma^0\Sigma^0)(\Xi^- p)} & G_{(\Sigma^0\Sigma^0)(\Sigma^+\Sigma^-)} & G_{(\Sigma^0\Sigma^0)(\Sigma^0\Sigma^0)} & G_{(\Sigma^0\Sigma^0)(\Sigma^0\Lambda)} & G_{(\Sigma^0\Sigma^0)(\Lambda\Lambda)} \\
G_{(\Sigma^0\Lambda)(\Xi^0 n)} & G_{(\Sigma^0\Lambda)(\Xi^- p)} & G_{(\Sigma^0\Lambda)(\Sigma^+\Sigma^-)} & G_{(\Sigma^0\Lambda)(\Sigma^0\Sigma^0)} & G_{(\Sigma^0\Lambda)(\Sigma^0\Lambda)} & G_{(\Sigma^0\Lambda)(\Lambda\Lambda)} \\
G_{(\Lambda\Lambda)(\Xi^0 n)} & G_{(\Lambda\Lambda)(\Xi^- p)} & G_{(\Lambda\Lambda)(\Sigma^+\Sigma^-)} & G_{(\Lambda\Lambda)(\Sigma^0\Sigma^0)} & G_{(\Lambda\Lambda)(\Sigma^0\Lambda)} & G_{(\Lambda\Lambda)(\Lambda\Lambda)}
\end{pmatrix}$$

Q=+1

$$\begin{pmatrix}
G^{SLJ}_{(\Xi^0 p)(\Xi^0 p)} & G_{(\Xi^0 p)(\Sigma^+\Lambda)} \\
G_{(\Sigma^+\Lambda)(\Xi^0 p)} & G_{(\Sigma^+\Lambda)(\Sigma^+\Lambda)}
\end{pmatrix}$$

Q=−1

$$\begin{pmatrix}
G^{SLJ}_{(\Xi^- n)(\Xi^- n)} & G_{(\Xi^- n)(\Sigma^-\Lambda)} \\
G_{(\Sigma^-\Lambda)(\Xi^- n)} & G_{(\Sigma^-\Lambda)(\Sigma^-\Lambda)}
\end{pmatrix}$$

Brueckner-Hartree-Fock

- ΞN G-matrix using $V^{\mathrm{LQCD}}_{S=-2}$, $M^{\mathrm{Phys}}_{N,Y}$, $U^{\mathrm{AV18}}_{n,p}$, $U^{\mathrm{LQCD}}_{\Lambda,\Sigma}$ and, U^{LQCD}_{Ξ}

Flavor anti-symmetric $^3S_1$, $^3D_1$ sectors

Q=0

$$\begin{pmatrix} G^{SLJ}_{(\Xi^0 n)(\Xi^0 n)} & G_{(\Xi^0 n)(\Xi^- p)} & G_{(\Xi^0 n)(\Sigma^+\Sigma^-)} & G_{(\Xi^0 n)(\Sigma^0\Lambda)} \\ G_{(\Xi^- p)(\Xi^0 n)} & G_{(\Xi^- p)(\Xi^- p)} & G_{(\Xi^- p)(\Sigma^+\Sigma^-)} & G_{(\Xi^- p)(\Sigma^0\Lambda)} \\ G_{(\Sigma^+\Sigma^-)(\Xi^0 n)} & G_{(\Sigma^+\Sigma^-)(\Xi^- p)} & G_{(\Sigma^+\Sigma^-)(\Sigma^+\Sigma^-)} & G_{(\Sigma^+\Sigma^-)(\Sigma^0\Lambda)} \\ G_{(\Sigma^0\Lambda)(\Xi^0 n)} & G_{(\Sigma^0\Lambda)(\Xi^- p)} & G_{(\Sigma^0\Lambda)(\Sigma^+\Sigma^-)} & G_{(\Sigma^0\Lambda)(\Sigma^0\Lambda)} \end{pmatrix}$$

Q=+1

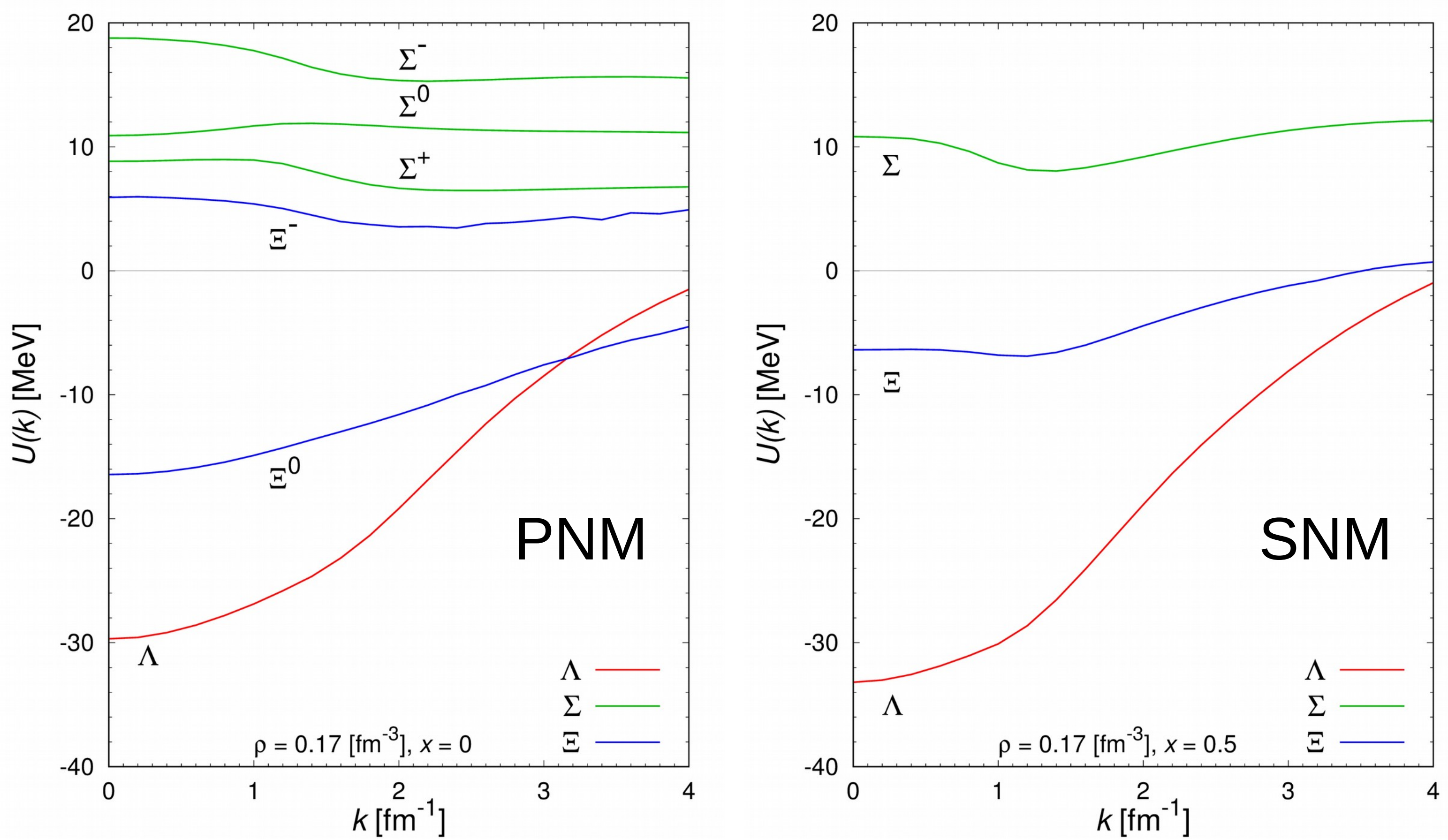
$$\begin{pmatrix} G^{SLJ}_{(\Xi^0 p)(\Xi^0 p)} & G_{(\Xi^0 p)(\Sigma^+\Sigma^0)} & G_{(\Xi^0 p)(\Sigma^+\Lambda)} \\ G_{(\Sigma^+\Sigma^0)(\Xi^0 p)} & G_{(\Sigma^+\Sigma^0)(\Sigma^+\Sigma^0)} & G_{(\Sigma^+\Sigma^0)(\Sigma^+\Lambda)} \\ G_{(\Sigma^+\Lambda)(\Xi^0 p)} & G_{(\Sigma^+\Lambda)(\Sigma^+\Sigma^0)} & G_{(\Sigma^+\Lambda)(\Sigma^+\Lambda)} \end{pmatrix}$$

Q=−1

$$\begin{pmatrix} G^{SLJ}_{(\Xi^- n)(\Xi^- n)} & G_{(\Xi^- n)(\Sigma^-\Sigma^0)} & G_{(\Xi^- n)(\Sigma^-\Lambda)} \\ G_{(\Sigma^-\Sigma^0)(\Xi^- n)} & G_{(\Sigma^-\Sigma^0(\Sigma^-\Sigma^0))} & G_{(\Sigma^-\Sigma^0)(\Sigma^-\Lambda)} \\ G_{(\Sigma^-\Lambda)(\Xi^- n)} & G_{(\Sigma^-\Lambda)(\Sigma^-\Sigma^0)} & G_{(\Sigma^-\Lambda)(\Sigma^-\Lambda)} \end{pmatrix}$$

Results

Hyperon single-particle potentials

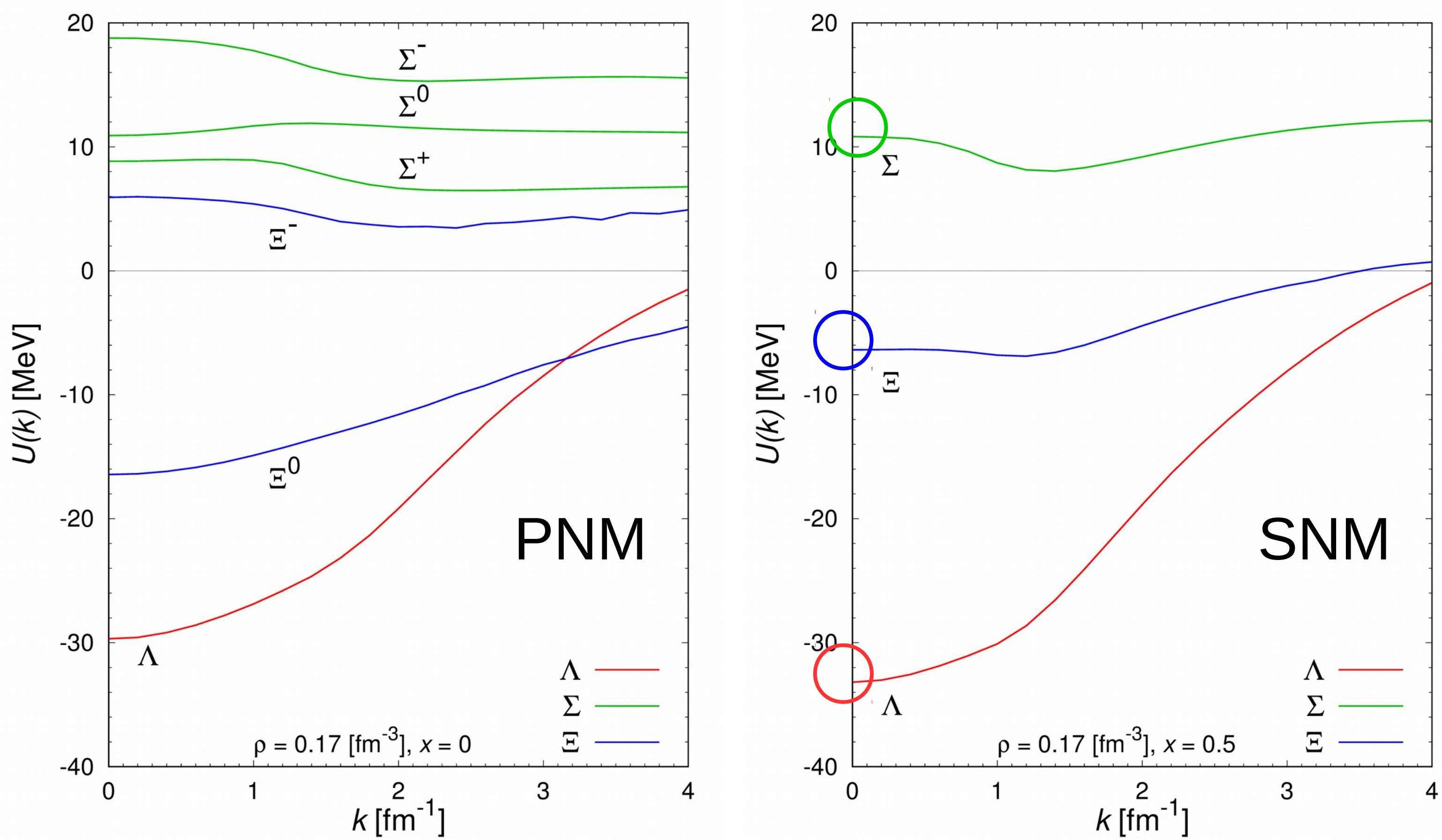


$@\rho=0.17\,[\mathrm{fm}^{-3}]$

Preliminary

- obtained by using YN,YY forces form QCD.

Hyperon single-particle potentials



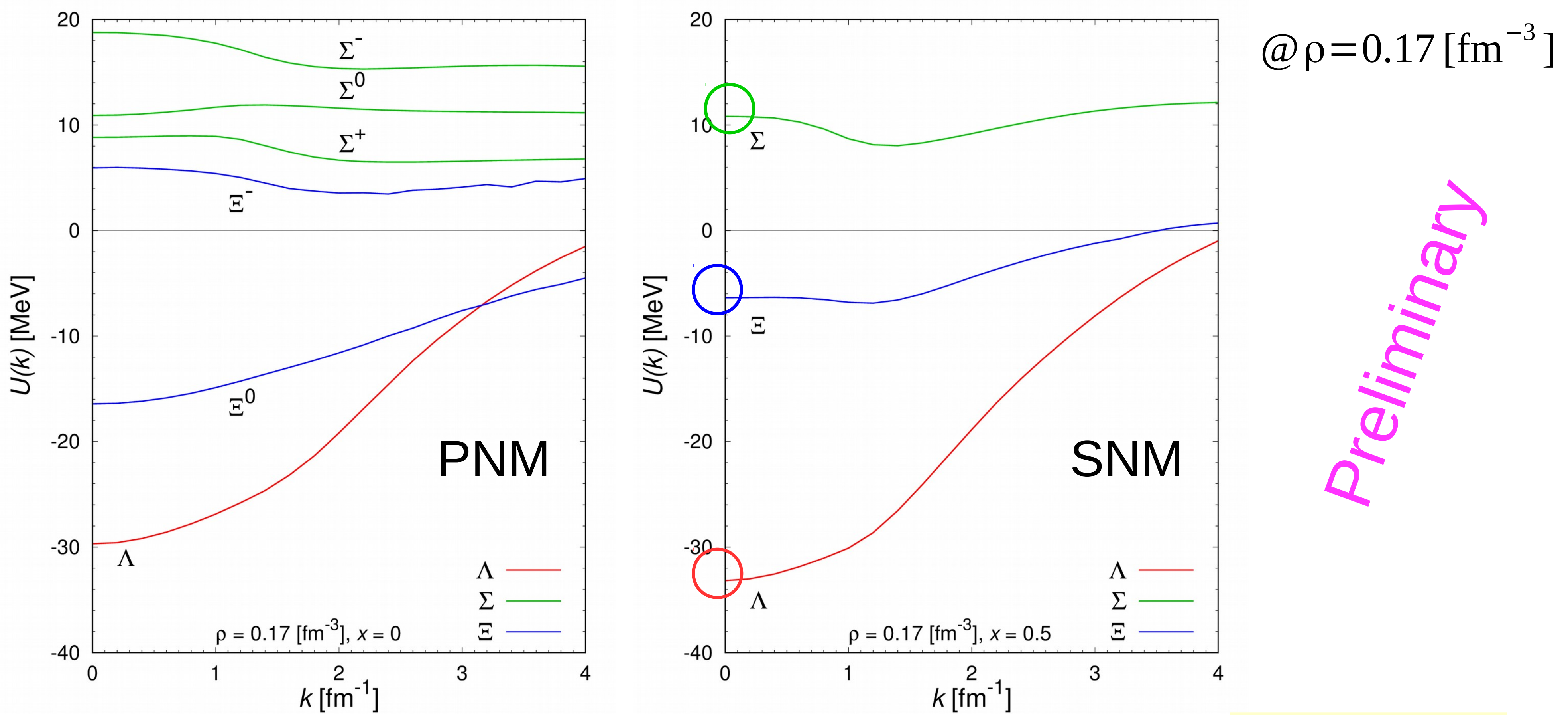
$@\rho = 0.17\,[\mathrm{fm}^{-3}]$

- obtained by using YN,YY forces form QCD.
- Results are compatible with experimental suggestion.

$$U_\Lambda^{\mathrm{Exp}}(0) \simeq -30, \quad U_\Xi(0)^{\mathrm{Exp}} \simeq -10, \quad U_\Sigma^{\mathrm{Exp}}(0) \geq +20 \quad [\mathrm{MeV}]$$

attraction　　attraction small　　repulsion

Hyperon single-particle potentials



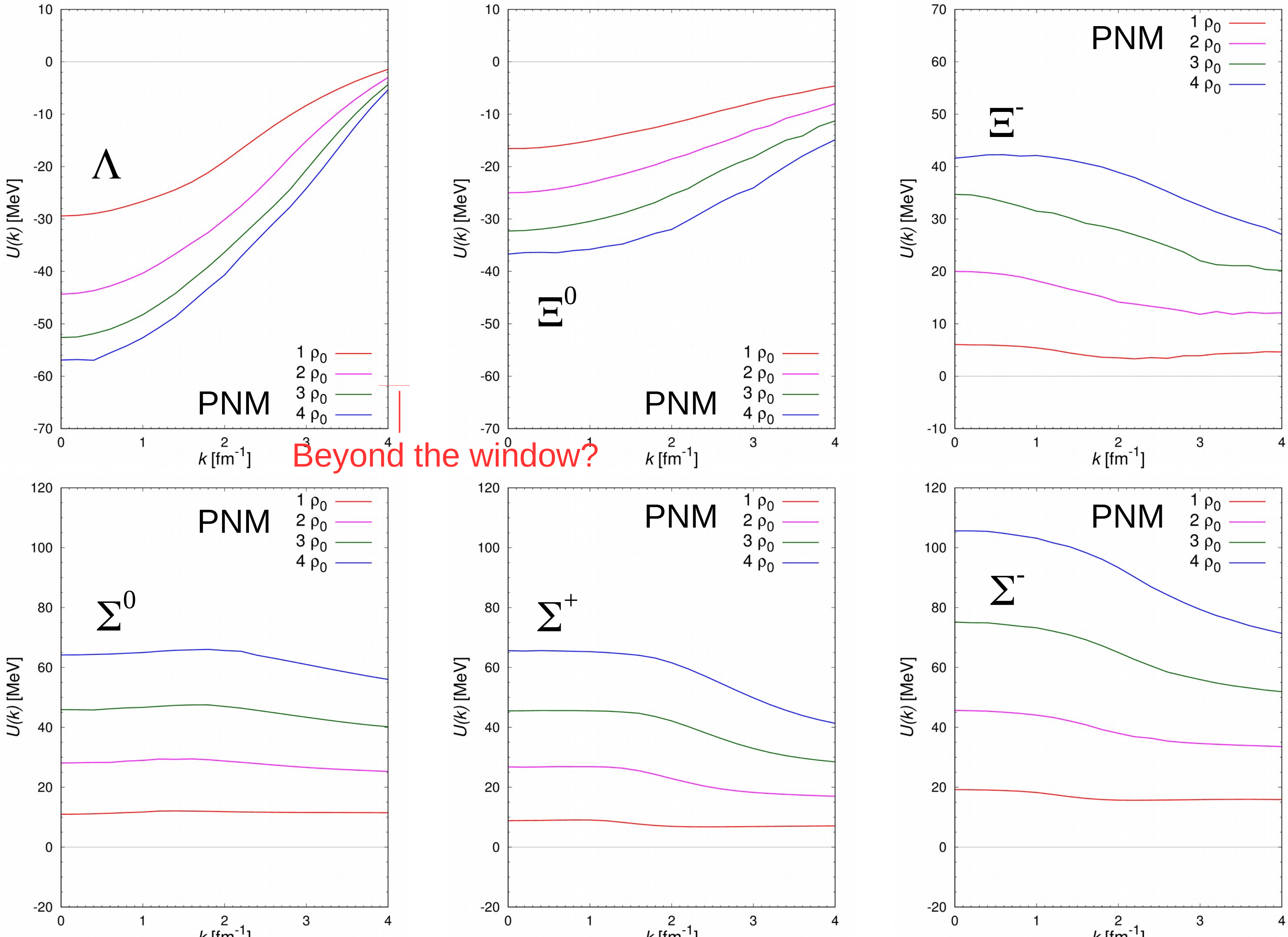
Remarkable.
Encouraging.

- obtained by using YN,YY forces form QCD.
- Results are compatible with experimental suggestion.

$$U_\Lambda^{\text{Exp}}(0) \simeq -30\,, \quad U_\Xi(0)^{\text{Exp}} \simeq -10\,, \quad U_\Sigma^{\text{Exp}}(0) \geq +20 \quad [\text{MeV}]$$

attraction　　attraction small　　repulsion

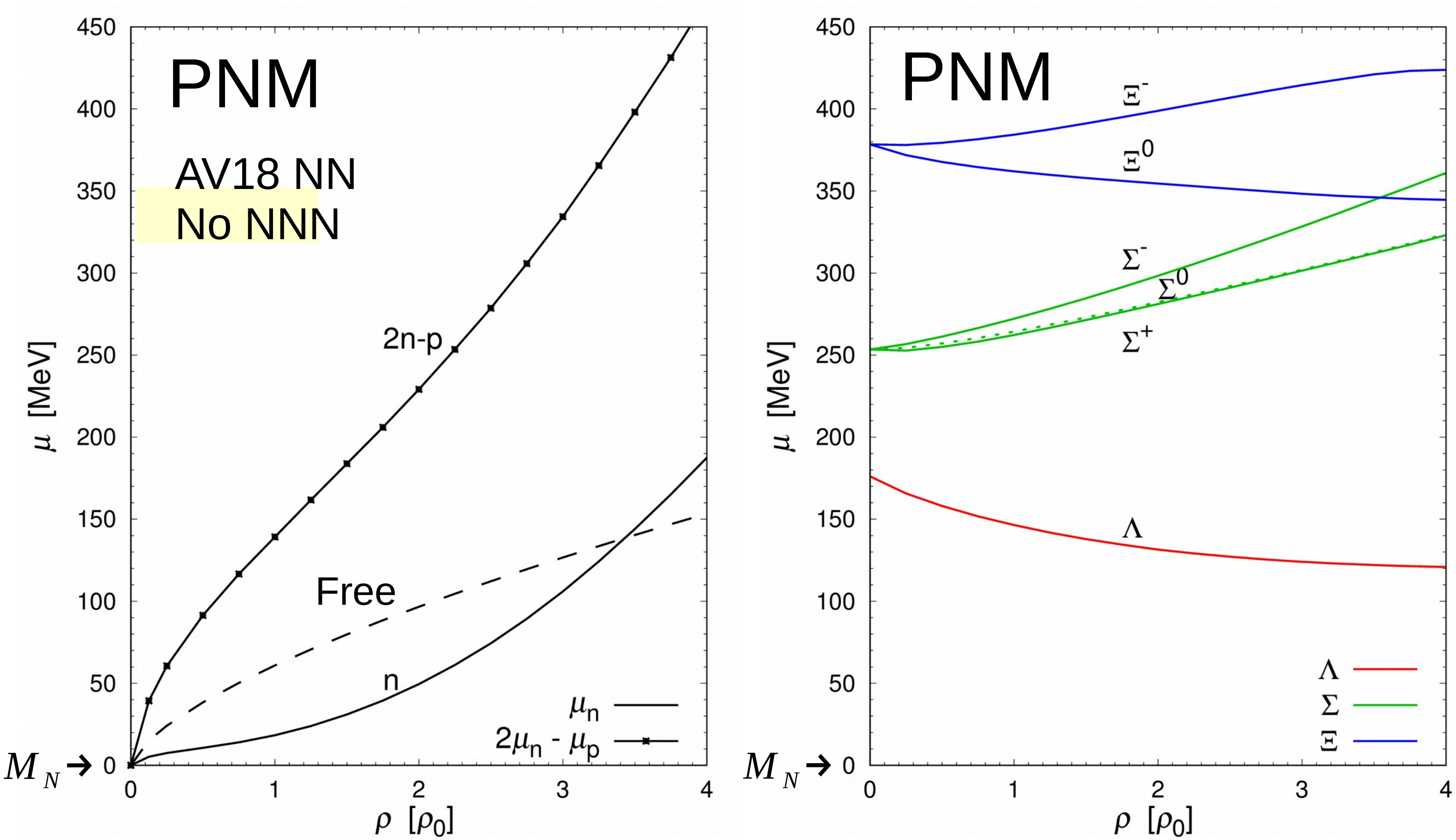
In high density PNM



S-wave
YN only

Preliminary

Chemical potentials



S-wave YN only

Preliminary

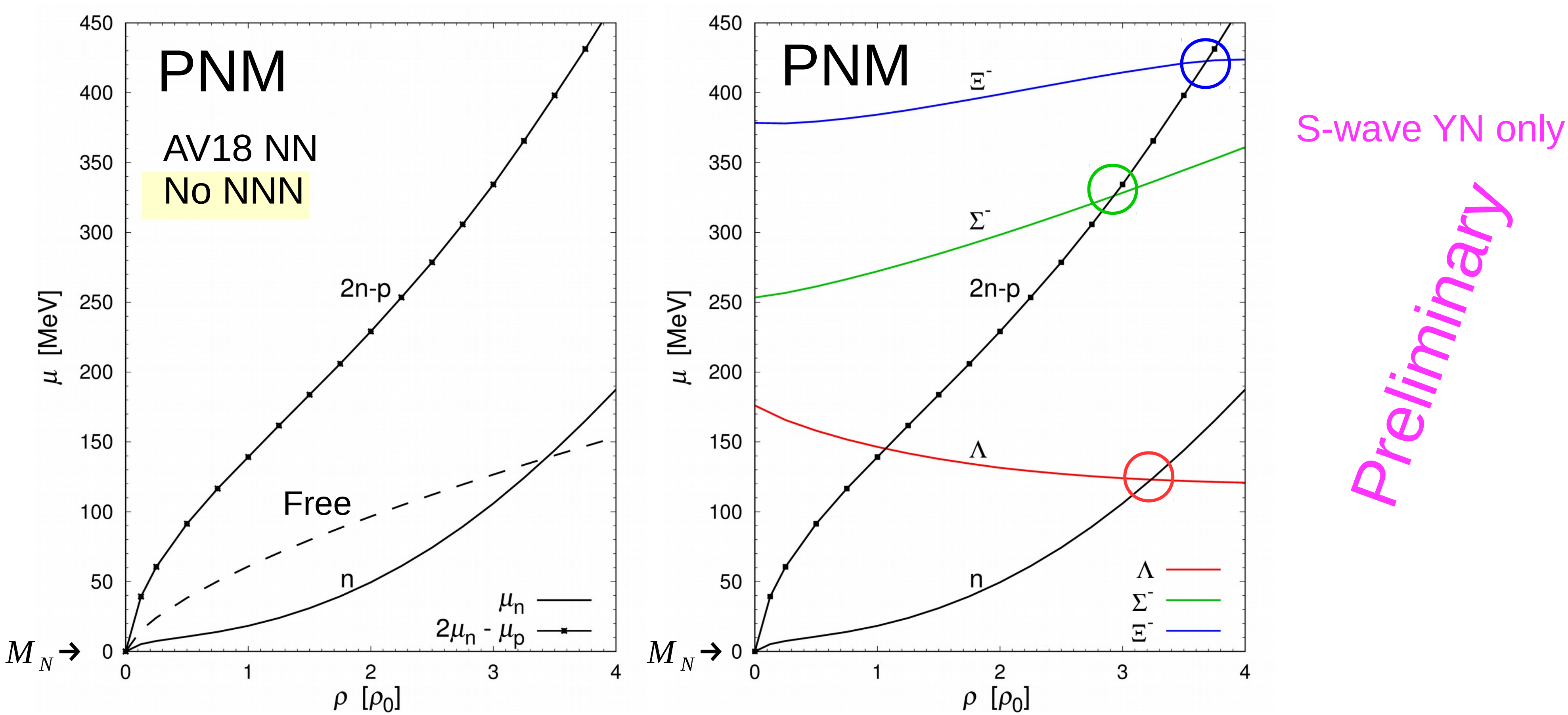
- Density dependence of chemical pot. of n and Y in PNM.

$$\mu_n(\rho) = \frac{k_F^2}{2M} + U_n(\rho; k_F), \quad \mu_Y(\rho) = M_Y - M_N + U_Y(\rho; 0)$$

- Hyperon appear as $n \rightarrow Y^0$ if $\mu_n > \mu_{Y^0}$

  $nn \rightarrow pY^-$ if $2\mu_n > \mu_p + \mu_{Y^-}$

Hyperon onset (just for a demonstration)



- First, Σ^- appear at 2.9 ρ_0. Next, Λ appear at 3.3 ρ_0.
  - NS matter is not PNM especially at high density.
  - We should compare with more sophisticated μ_n and μ_p.
  - P-wave YN force may be important at high density.

Summary and Outlook

Summary and Outlook

- We've explained our goal and approach
  - Want to do (strange) nuclear physics starting from QCD.
  - Extract BB interaction potentials in lattice QCD simulation.
  - Apply potentials to few-body technique or many-body theory.
  - Unique way to do nuclear physics based on QCD, I think.
- We've introduced HALQCD method
  - Utilize 4-pt function containing information of the interaction.
  - This is a unique solution to the serious plateau crisis in the direct LQCD method for multi-hadron system.
- We've shown HALQCD BB potentials
  - We obtain QCD prediction of hyperon interactions.
  - We obtain (qualitatively) reasonable two-nucleon force.
  - We reveal nature of general BB S-wave interactions
    - which prove that quark cluster model prediction is correct.

Summary and Outlook

Resuls of application

- We calculated hyperon s.p. potentials w/ the prediction.
  - This time, I used rotated data diagonal in the irre.-rep. base.
- We obtained U_Y compatible with experiment!
  - In SNM, Λ and Ξ feel attracsion, while Σ feels repulsion.
- This is remarkable success, at least encouraging.
  - Recall that we've never used any experimental data about hepron interactions, but we used only QCD.

Outlook

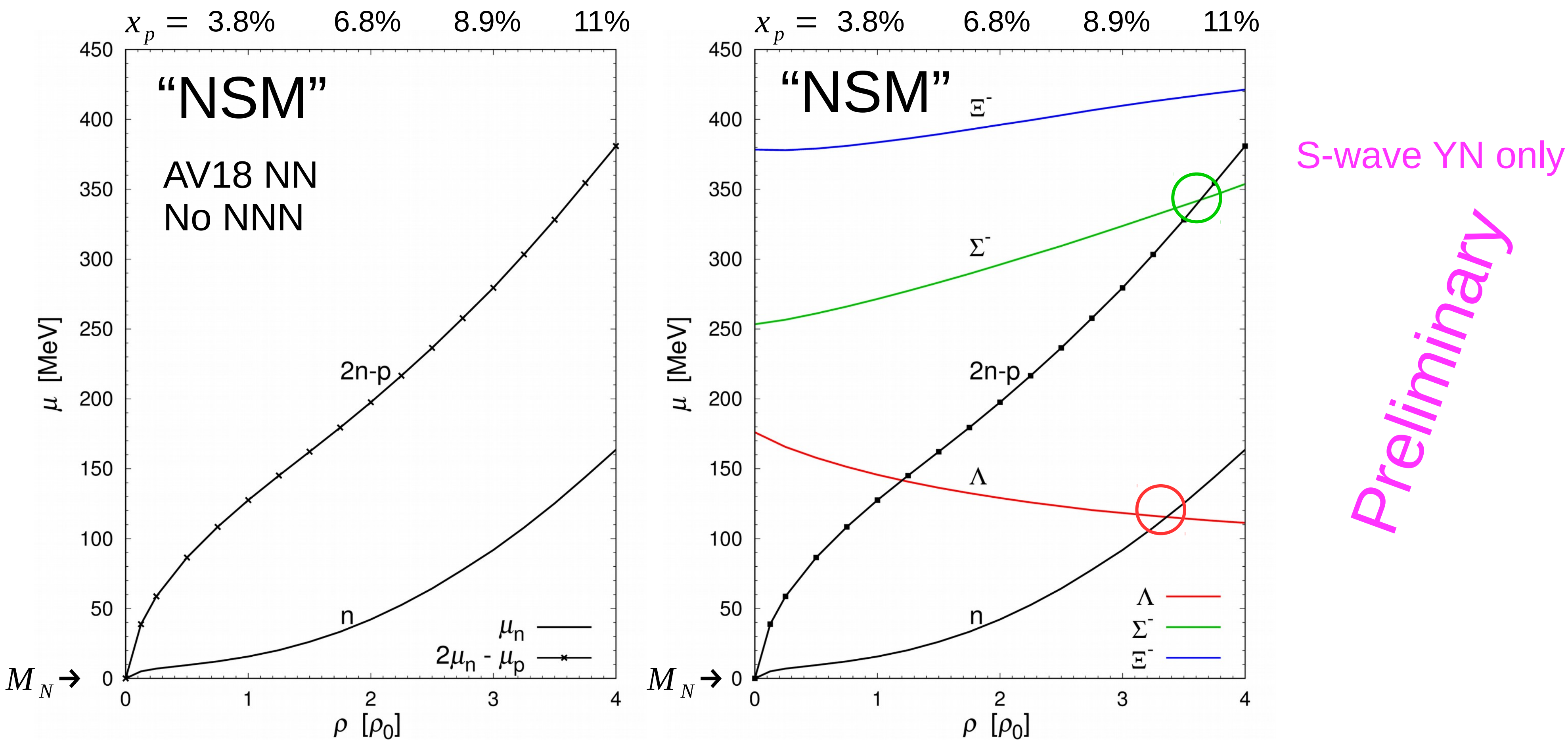
- We'll use original data w/ the physical $SU(3)_F$ breaking.
- We'll estimate statistical error & systematic uncertainty. and try to reduce them.
- We'll include hyperon interactions in higher p. w.

Next generation Super-computer

- We will be able to explain data of hypernuclei from QCD and hope we can reveal hyperons in NS, in near future.

Thank you !!

Hyperon onset (just for a demonstration)



- "NSM" is matter w/ n, p, e, μ under β-eq and Q=0.