The 3-3-1 Models and New Physics

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Content

I. Introduction

1. Standard Model (SM) of strong and electroweak interactions

- Strong interactions -Interactions of coloured quarks with gluons based on $SU(3)_C$

  - Asymptotic freedom (Theory of strong interaction based on $SU(3)_C$ gauge group has AF.

  - Confinement?
- Glashow-Weinberg-Salam model of EM and weak interactions based on $SU(2)_L \otimes U(1)_Y$
  - Leptons and quarks are separated into 3 generations

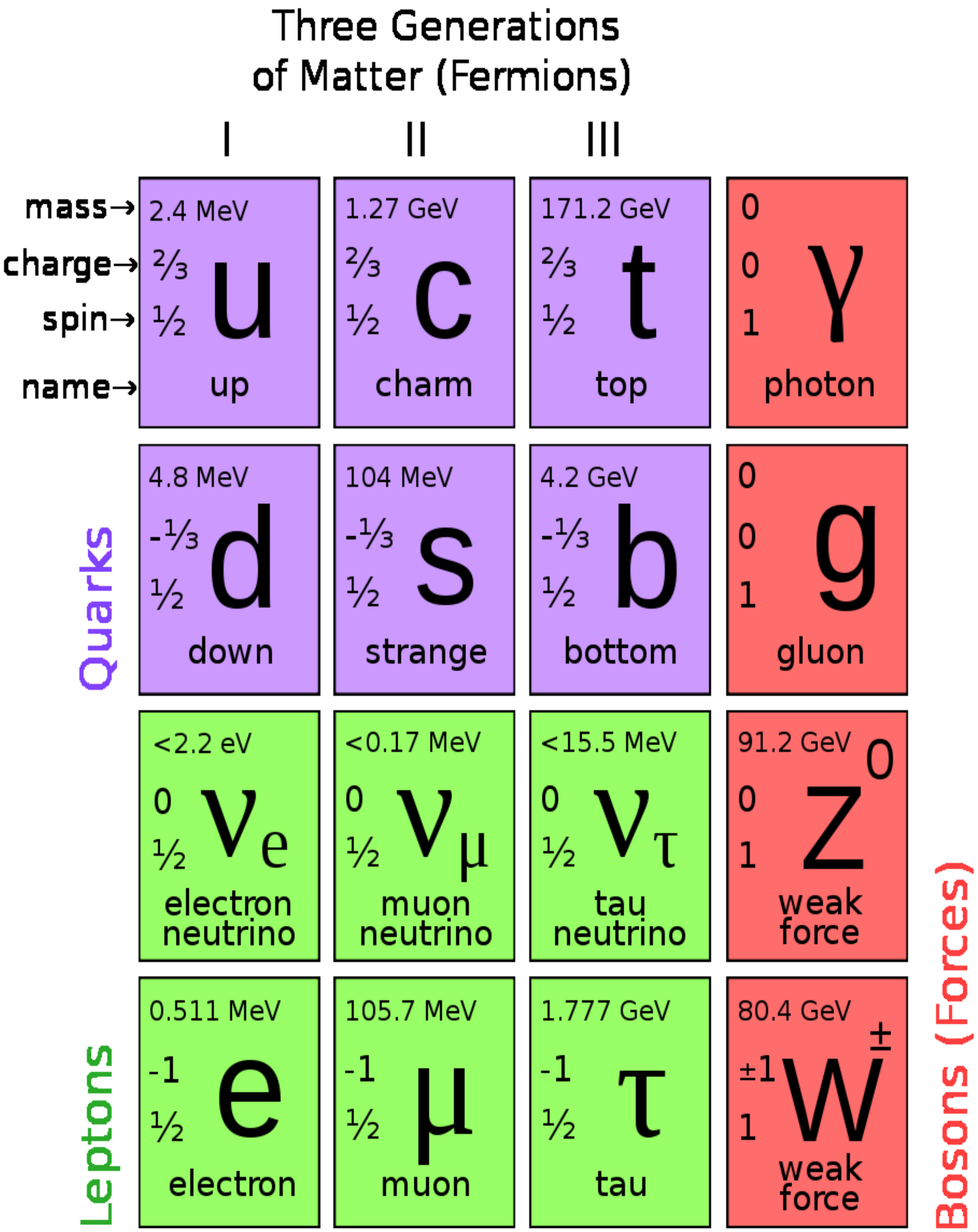
  - Why 3?

1. The Standard Model based on gauge group $\mathrm{SU(3)}_C \otimes \mathrm{SU(2)}_L \otimes \mathrm{U(1)}_Y$ is great success of Physics in the 20^{th} Century.
2. Neutrinos are massive with tiny masses (SuperKamiokande (1998) - the first and unique evidence for New Physics)
3. New physics (NP) = everything $\neq$ SM = Lepton (flavor) number violation, Dark Matter, BAU

   New physics = MSSM, left-right symmetry, 3-3-1 models ???
4. LHC confirms: Higgs boson with mass ~ 125 GeV

   Questions:

   - Higgs boson with mass 125 GeV is the SM one?
   - It belongs to beyond SM models?
   - One fundamental question: Number of fermion generations?: 3, 4, ...The 3-3-1 models give very nice answer.



Three Generations
of Matter (Fermions)

	I	II	III	
mass→ charge→ spin→ name→	2.4 MeV ⅔ ½ u up	1.27 GeV ⅔ ½ c charm	171.2 GeV ⅔ ½ t top	0 0 1 γ photon
Quarks	4.8 MeV -⅓ ½ d down	104 MeV -⅓ ½ s strange	4.2 GeV -⅓ ½ b bottom	0 0 1 g gluon
	<2.2 eV 0 ½ ν_e electron neutrino	<0.17 MeV 0 ½ ν_μ muon neutrino	<15.5 MeV 0 ½ ν_τ tau neutrino	91.2 GeV 0 1 Z^0 weak force
Leptons	0.511 MeV -1 ½ e electron	105.7 MeV -1 ½ μ muon	1.777 GeV -1 ½ τ tau	80.4 GeV ±1 1 $W^\pm$ weak force

Bosons (Forces)

The beyond Standard Model Some directions for extension of the SM are

- 4 D space-time is extended by anti-commuting Grassmann variables $\theta \Rightarrow$ Minimal Supersymmetric Standard Model (MSSM).
  - ⋆ In SUSY models: the Higgs self-couplings can be determined.
- Extra-dimensions: 4D → 5D, 6D (Hosotani models)
  - ⋆ Higgs fields are gauge fields in extra dimensions
- Adding new particle content: new Higgs doublet ⇒ two Higgs doublet model,...
- Extension by flavor group: SM $SU(2)_L \otimes U(1)_Y \Rightarrow SU(2)_R \otimes SU(2)_L \otimes U(1)_Y$. Then left-right symmetry model. $SO(10)$ model, ...
- $SU(2)_L \otimes U(1)_Y \Rightarrow SU(3)_L \otimes U(1)_X$. Then ones get $SU(3)_C \otimes SU(3)_L \otimes U(1)_Y$ We call 3-3-1 model for short.

- Number of generations $N_g = n \times N_c$= 3, 6, 9,...

- Asymptotic freedom $N_g < 5 \quad \Rightarrow N_g = 3!!!$

- 3rd generation of quarks transforms differently from two others ⇒ top quark is very heavy.

II. The 3-3-1 models

The leptons lie in fundamental representation: triplet (**3**)

$$f_{aL} = \begin{pmatrix} \nu_a \\ l_a \\ L_a \end{pmatrix}_L \sim \mathbf{3}, \quad a = e, \mu, \tau \tag{1}$$

In Eq.(1) if

$L_a = l_a^c$ minimal version (Frampton-Pisano-Pleitez model)
$L_a = \nu_a^c$ 3-3-1 model with right-handed neutrinos (Foot-Long-Tran model)

The electric charge operator is in the form

$$Q = T_3 + \beta T_8 + X, \tag{2}$$

where $\beta = \pm\sqrt{3}$ corresponds to the minimal version, while $\beta = \pm 1/\sqrt{3}$ - the version with right-handed neutrinos.

III. The minimal 3-3-1 model

F. Pisano, V. Pleitez, PRD (1992); P. Frampton, PRL (1992)

0.1 Particle content

-Leptons in triplet

$$L_a = \begin{pmatrix} l_a \\ -\nu_a \\ l_a^c \end{pmatrix}_L \sim (\mathbf{3}, \tilde{\mathbf{3}}, \mathbf{0}), \quad (l_a^c)_L \equiv (l_{aR})^c \tag{3}$$

Note: No right-handed charged lepton in singlet. Therefore lepton masses is generated by unusual way!

The electric charge operator is given by

$$Q = T_3 + \sqrt{3} T_8 + X, \tag{4}$$

- Two quark generations in antitriplets and one in triplet

$$Q_{\alpha L} = \begin{pmatrix} u_{\alpha L} \\ d_{\alpha L} \\ D_{\alpha L} \end{pmatrix} \sim (\mathbf{3}, \mathbf{3}, \mathbf{0}), \ \alpha = 1, 2, \tag{5}$$

$$Q_{3L} = \begin{pmatrix} b \\ -t \\ T \end{pmatrix}_L \sim (\mathbf{3}, \tilde{\mathbf{3}}, \mathbf{1/3}), T_R \sim (\mathbf{3}, \mathbf{1}, \mathbf{5/3}), \tag{6}$$

$$D_{\alpha R} \sim (\mathbf{3}, \mathbf{1}, \mathbf{-4/3}). \tag{7}$$

To break spontaneously symmetry this model, we need three triplets and one sextet of Higgs fields

$$\begin{aligned}
\Phi &= \left(\phi^{++}, \phi^{+}, \phi^{,0}\right)^T \sim (1,3,1), \qquad (8)\\
\Delta &= \begin{pmatrix} \Delta_1^+ \\ \Delta^o \\ \Delta_2^- \end{pmatrix} \sim (1,3,0),\\
\Delta' &= \left(\Delta'^0, \Delta'^-, \Delta'^{--}\right)^T \sim (1,3,-1),\\
\eta &= \begin{pmatrix} \eta_1^{++} & \eta_1^+/\sqrt{2} & \eta^o/\sqrt{2} \\ \eta_1^+/\sqrt{2} & \eta'^o & \eta_2^-/\sqrt{2} \\ \eta^o/\sqrt{2} & \eta_2^-/\sqrt{2} & \eta_2^{--} \end{pmatrix} \sim (1,6,0).
\end{aligned}$$

with VEV: $\langle \Delta^o \rangle = v/\sqrt{2}, \langle \Delta'^o \rangle = v'/\sqrt{2}$ and $\langle \eta^o \rangle = \omega/\sqrt{2},\ \langle \eta'^o \rangle = 0$.

The lepton masses arise from two Yukawa couplings

$$L_{Yukawa} = h_t \bar{f}_{aL} f^c_{aL} \Delta + h_s \bar{f}_{aL} \eta f_{aL} + H.c.\,, \tag{9}$$

where the first term is in antisymmetric form, while the second one is in symmetric form.

The first term provides antisymmetric 3 × 3 matrix, so one charged lepton is still massless. That is why we need to introduce sextet for the second term.

Gauge bosons

There are 9 gauge bosons:

- 4 of SM: $\gamma, Z, W^\pm$
- 5 new ones: $Z', Y^\pm, X^{\pm\pm}$

  The last two are bilepton gauge bosons carrying lepton number 2.
- The singly charged bilepton $Y^\pm$ is responsible for the wrong muon decay (NP effect)

  $$\mu^- \to e^- + \nu_e + \tilde{\nu_\mu}, \quad < 1.2 \quad \% \quad \text{with 95 CL} \tag{10}$$

  Process (10) gives a lower bound: $M_Y \leq 230$ GeV
- Doubly charged bileptons $X^{\pm\pm}$ lead to special four charged lepton final state

  $$\begin{aligned} e^- + e^+ &\to X^{--} + X^{++} \\ &\to l_a^- + l_a^- + l_b^+ + l_b^+\,, a, b = e, \mu, \tau \quad \text{special process} \end{aligned}$$
- Exotic quarks T, D_α have electric charges 5/3 and -4/3, respectively, and carry lepton number 2.


μ^-
W
e^-
$\tilde{\nu}_e$
ν_μ
μ^-
Y
e^-
ν_e^c
ν_μ^c

IV. The 3-3-1 with RH neutrinos

Foot, HNL, Tuan A. Tran, PRD (1994); HNL, PR D53, PR D54 (1996)

0.2 Particle content

-Leptons in triplet

$$f^a_L = \begin{pmatrix} \nu^a_L \\ l^a_L \\ \nu^{ca}_L \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, -\mathbf{1}/\mathbf{3}), l^a_R \sim (1, 1, -1), \tag{11}$$

where $a = 1, 2, 3$

- Two quark generations in antitriplets and one in triplet

$$Q_{\alpha L} = \begin{pmatrix} d_{\alpha L} \\ -u_{\alpha L} \\ D_{\alpha L} \end{pmatrix} \sim (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{0}), \ \alpha = 1, 2, \tag{12}$$

$$Q_{3L} = \begin{pmatrix} t \\ b \\ T \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{3}, \mathbf{1}/\mathbf{3}), T_R \sim (3, 1, 2/3), \tag{13}$$

$$D_{\alpha R} \sim (3, 1, -1/3). \tag{14}$$

The exotic quarks T, D_α have electric charges the same as ordinary quarks. Hence this kind of model is also called by the model without exotic charges.

0.3 Higgs sector

For SSB, we need three Higgs triplets

$$\begin{aligned} \rho &= \begin{pmatrix} \rho_1^+ \\ \rho_2^0 \\ \rho_3^+ \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, \mathbf{2/3}), \quad \eta = \begin{pmatrix} \eta_1^0 \\ \eta_2^- \\ \eta_3^0 \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, -\mathbf{1/3}), \\ \chi &= \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \chi_3^0 \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, -\mathbf{1/3}). \end{aligned} \tag{15}$$

Note that η and χ have the same quantum number, but the difference is that their components have different structure:

$$\langle\rho\rangle = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}, \quad \langle\eta\rangle = \begin{pmatrix} u \\ 0 \\ 0 \end{pmatrix}, \quad \langle\chi\rangle = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix}.$$

Since lepton and anti-lepton lie in the same triplet, therefore lepton number in these models are not conserved. The new conserved value should have the form [D. Chang & HNL, PRD (2006)]

$$L = \alpha T_3 + \beta T_8 + \mathcal{L}. \tag{16}$$

Applying for the lepton triplet, we get

$$L = \frac{4}{\sqrt{3}}\lambda_8 + \mathcal{L} \tag{17}$$

Fields	ν_L^{ca}	l_L	l_R	ρ_3^+	η_3^0	χ_1^0	χ_2^-	$D_{\alpha L}$	$D_{\beta L}$	T_L	T_R
L	-1	1	1	-2	-2	2	2	2	2	-2	-2

- Higgs boson production at LHC $pp \to hZ$ was considered [Le Duc Ninh, HNL, PRD (2005)]

- Neutrinos get masses and have pseudo-Dirac inverted hierarchy mass pattern [D. Chang & HNL, PRD (2006)].

Question: Does the 3-3-1 model have disadvantages?

Answer: Higss sector is complicated (with 3 triplet)

- In 2006, the 3-3-1 model with two Higgs triplets has been constructed, and

IV. B meson anomalies and the 3-3-1 model

In the SM there is a generation replica (three generations are completely similar which means lepton number is universal). However, recent data: BABAR (2013), Belle (2015) and LHCb(2015) shows

$$\mathcal{R}(D^{(*)}) = \left.\frac{\mathcal{B}(\bar{B} \to D^{(*)}\tau^- \bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \to D^{(*)} l^- \bar{\nu}_l)}\right|_{l \in \{e,\mu\}},$$

where the SM predictions are given by $\mathcal{R}(D)_{SM} = 0.300 \pm 0.008$ and $\mathcal{R}(D^*)_{SM} = 0.252 \pm 0.003$, respectively. However, the present experimental values measured by BaBar (2013), Belle (2015) and LHCb (2015) collaborations have recently observed anomalies in the ratios $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$. The average values given by the Heavy Flavor Average Group (HFAG) (2015) are $\mathcal{R}(D)_{avg} = 0.397 \pm 0.040 \pm 0.028$ and $\mathcal{R}(D^*)_{avg} = 0.316 \pm 0.016 \pm 0.010$, which exceed the SM predictions by 1.9σ and 3.3σ, respectively. If one takes into account the $\mathcal{R}(D)$-$\mathcal{R}(D^*)$ correlation of -0.21, the tension with the SM expectation would be at 4σ level (C. S. Kim et al, PRD 95 (2017)

These anomalies have caused wide concern, leading to many works. The works in NP models are classified as mediated by leptoquarks, vector-like fermion, charged Higgs bosons, etc

⋆ The 3-3-1 model for $\mathcal{R}(D^{(*)})$ anomalies

[W. Ma, C-X Yue, PRD 95 (2017) 035040)]

The quark content is generally described by

$$
\begin{aligned}
q_{\alpha L} &= \begin{pmatrix} u_\alpha \\ -d_\alpha \\ D_\alpha \end{pmatrix}_L \sim (3^*, 3, 0),\\
q_{3L} &= \begin{pmatrix} u_3 \\ d_3 \\ T_3 \end{pmatrix}_L \sim (3, 3, 1/3),\\
&d^c \sim (3^*, 1, 1/3),\ u^c \sim (3^*, 1, -2/3),\\
&T^c \sim (3^*, 1, -2/3),\ D^c_\alpha \sim (3^*, 1, 1/3),
\end{aligned}
$$

For the leptonic sector, each lepton family is arranged in triplets: the first two elements are the charged and neutral lepton, and the third element is a conjugate of the charged lepton or the neutral lepton. There are

$$\begin{aligned}
\Psi_{nL} &= \begin{pmatrix} e_n^- \\ \nu_n \\ N_n^0 \end{pmatrix}_L \sim (1, 3^*, -1/3), \\
\Psi_L &= \begin{pmatrix} \nu_1 \\ e_1^- \\ E_1^- \end{pmatrix}_L \sim (1, 3, -2/3), \\
\Psi_{4L} &= \begin{pmatrix} E_2^- \\ N_3^0 \\ N_4^0 \end{pmatrix}_L \sim (1, 3^*, -1/3), \\
\Psi_{5L} &= \begin{pmatrix} N_5^0 \\ E_2^+ \\ e_3^+ \end{pmatrix}_L \sim (1, 3^*, 2/3), \\
& e_n^c \sim (1,1,1), \quad e_3^c \sim (1,1,1), \\
& E_1^c \sim (1,1,1), \quad E_2^c \sim (1,1,1),
\end{aligned}$$

where $n = 2, 3$.

There are two extra lepton anti-triplets.

Besides the ordinary gauge bosons γ, Z and $W^{\pm}$, new neutral gauge boson Z', Higgs sector content

$$\Phi_1 = \begin{pmatrix} \phi_1^0 \\ \phi_1^+ \\ \eta_1^+ \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^- \\ \phi_2^0 \\ \eta_2^0 \end{pmatrix}, \quad \varphi = \begin{pmatrix} \eta_3^- \\ \eta_3^0 \\ \phi_3^0 \end{pmatrix}, \tag{18}$$

We have the physical singly-charged Higgs bosons $H^\pm$, the CP-odd Higgs boson A and the CP-even Higgs boson H_a [26]. The contributions from the 3-3-1 models to the $\mathcal{R}(D^{(*)})$ and $\mathcal{R}_\tau(D^{(*)})$ anomalies may induced both by the charged Higgs bosons $H^\pm$ and the new gauge boson W'. However, the mass of the new gauge boson W' is big, and the interactions between W' and the SM particles are faint. Comparing with the contribution from the SM gauge boson W, the contribution from W' is tiny enough to be ignored. Therefore, we will concentrate on calculating the contributions from the charged Higgs bosons $H^\pm$ in the following parts.

The Yukawa coupling for the charged Higgs boson H^+ from the 3-3-1 models is generally given by [Alves et al PRD 84 (2011), S. M. Boucenna et al, PRD 90 (2014)]

$$\begin{aligned} G_{\bar{u}dH^+} \; = \; & -\frac{g}{2\sqrt{2}m_W}\left[V_{CKM} - V_L^{u\dagger}\Delta V_L^d\right]^*_{ud}[m_d(1 \\ & -\gamma_5)tan\beta + m_u(1+\gamma_5)cot\beta], \end{aligned} \tag{19}$$

where u and d refer to up- and down-type quarks, m_W, m_u and m_d express the masses of the SM particles W, u and d, $\Delta = diag(0,0,1)$. V_L^u and V_L^d are the unitary matrices of the left-handed quarks, which are given by [26,27]

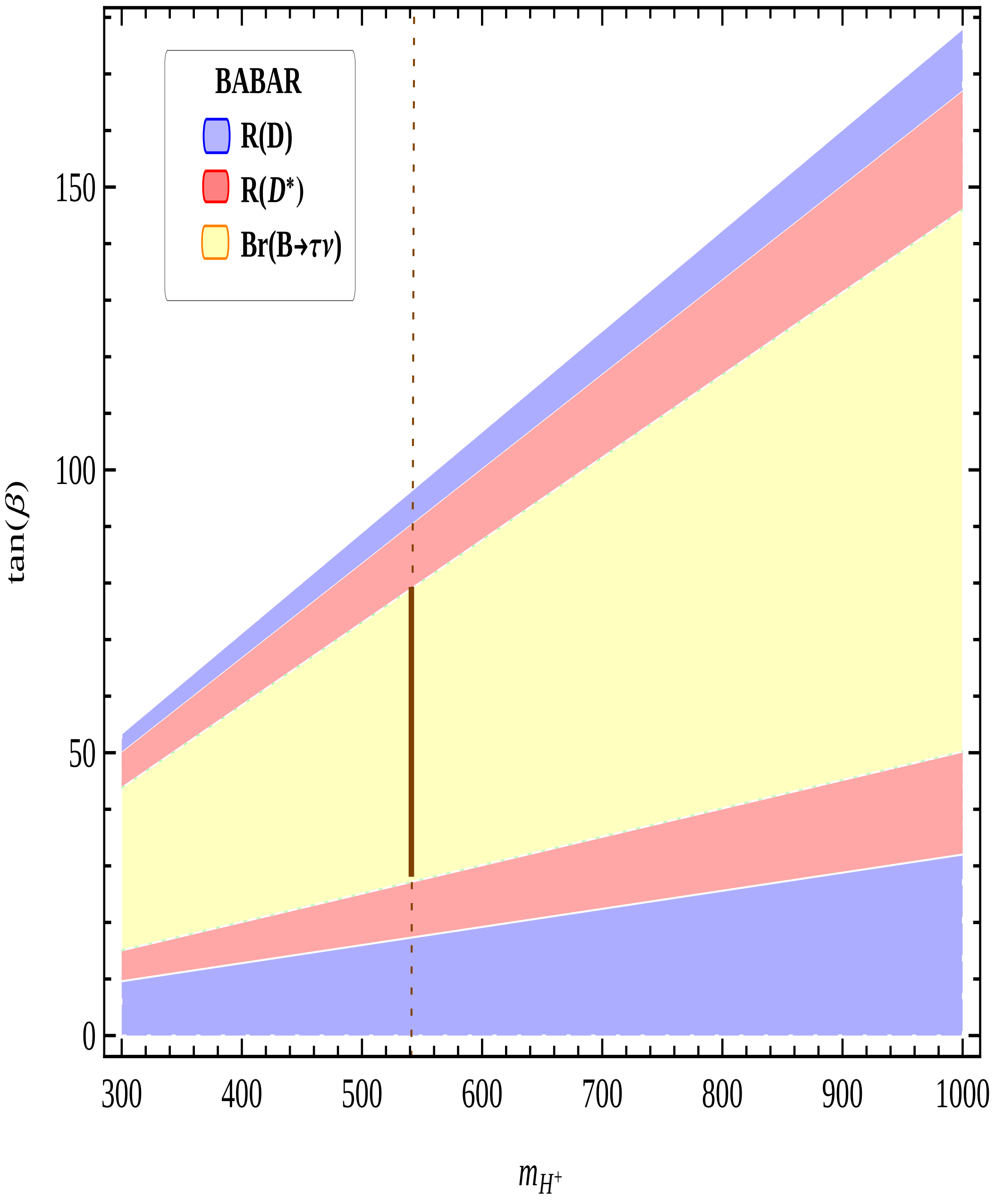
$$V_L^u = \begin{pmatrix} 0.975 & -0.223 & 1.86\times 10^{-3} \\ 0.222 & 0.974 & 0.0518 \\ -0.01340 & -0.0501 & 0.999 \end{pmatrix}, \tag{20}$$

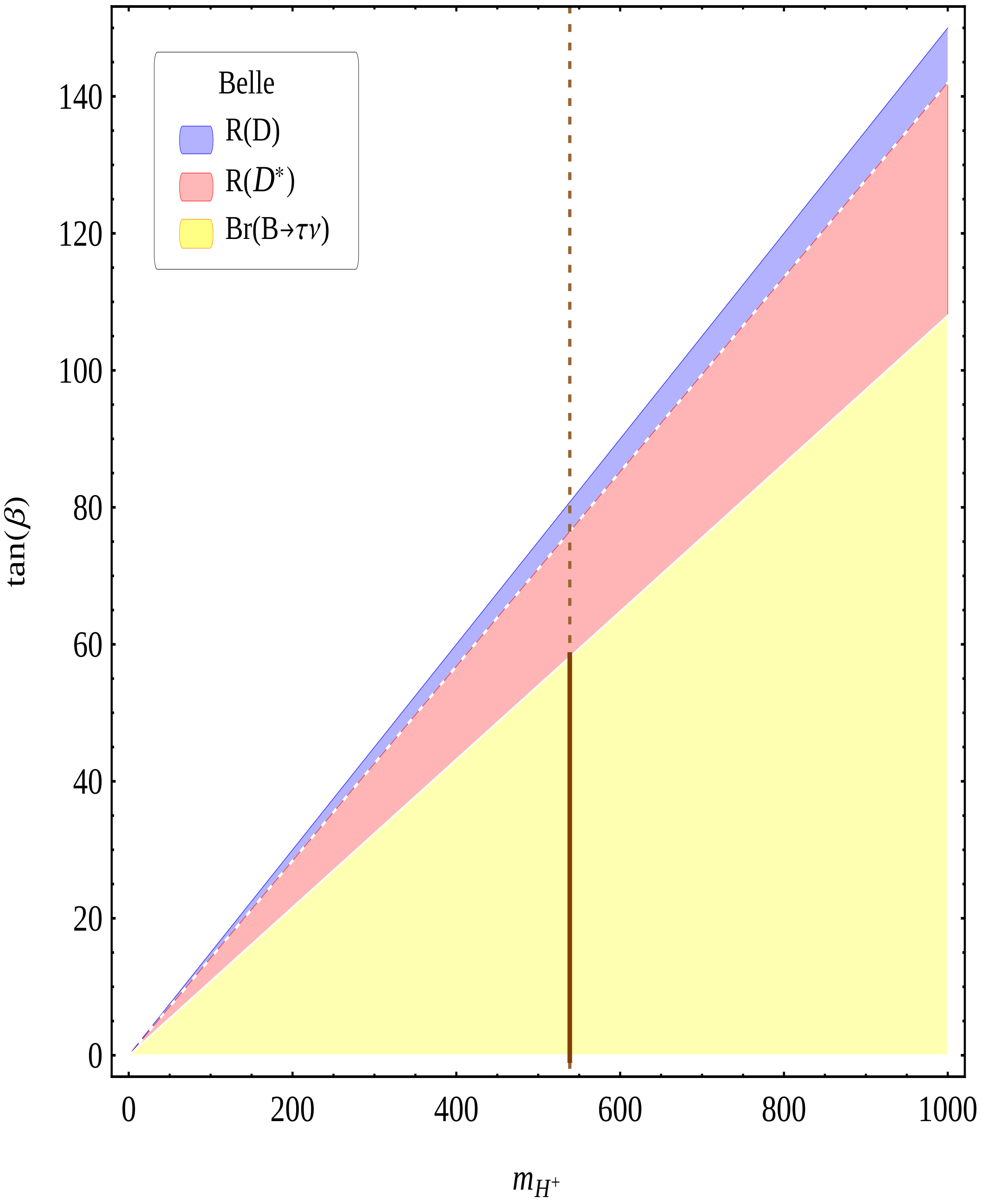
$$V_L^d = \begin{pmatrix} 1.00 & 2.56\times 10^{-3} & 5.87\times 10^{-3} \\ -3.10\times 10^{-3} & 0.996 & 0.0941 \\ -5.61\times 10^{-3} & -0.0942 & 0.996 \end{pmatrix}, \tag{21}$$

where $tan\beta = v_2/v_1$.

The ratios $\mathcal{R}_\tau^{331}(D^{(*)})$ can be written as

$$\begin{aligned}\mathcal{R}_\tau^{331}(D^{(*)}) &= \frac{\mathcal{R}^{331}(D^{(*)})}{\mathcal{B}(B^+ \to \tau^+\nu_\tau)} \\ &\approx \mathcal{R}_\tau^{SM}(D^{(*)}) - \frac{A_{D^{(*)}}}{\mathcal{B}(B^+ \to \tau^+\nu_\tau)} \cdot \frac{tan^2\beta}{m_{H^+}^2} \\ &\quad + \frac{B_{D^{(*)}}}{\mathcal{B}(B^+ \to \tau^+\nu_\tau)} \cdot \frac{tan^4\beta}{m_{H^+}^4}, \end{aligned} \qquad (22)$$


BABAR
R(D)
R(D^*)
Br(B→τν)
tan(β)
0
50
100
150
300
400
500
600
700
800
900
1000
m_{H^+}


Belle
R(D)
R(D^*)
Br(B→$\tau\nu$)
tan(β)
m_{H^+}
0
20
40
60
80
100
120
140
0
200
400
600
800
1000

Another topic related to lepton flavor number violating delay of neutral Higgs boson $H \to \mu^{\pm}\tau^{\mp}$ was considered in works P. T. Giang, L. T. Hue, D. T. Huong and HNL *Nucl. Phys.* **B 864**, (2012) 85 – 112; T. T. Thuc, L. T. Hue, HNL, T. Phong Nguyen:, *Phys. Rev.* D **93**, 115026 (2016).

V. Conclusions

1. Problems in neutrino physics:
    - Absolute values of masses? Waiting for neutrino less double beta decays result.
    - The θ_{13} mixing: solar neutrino, reactor data, Smirnov-Mikheyev resonance
    - Flavor discrete symmetry?
2. Higgs physics at LHC
    - Decays $H \to \gamma\gamma$, $H \to Z\gamma$
    - Rare decays of tau, Z and Z' bosons
    - Electroweak phase transition in the 3-3-1 model (V.Q. Phong, HNL, V. T. Van, PRD 2014, EPJC 2015.