MICROSCOPIC DESCRIPTION OF AVERAGE LEVEL SPACING IN EVEN - EVEN NUCLEI

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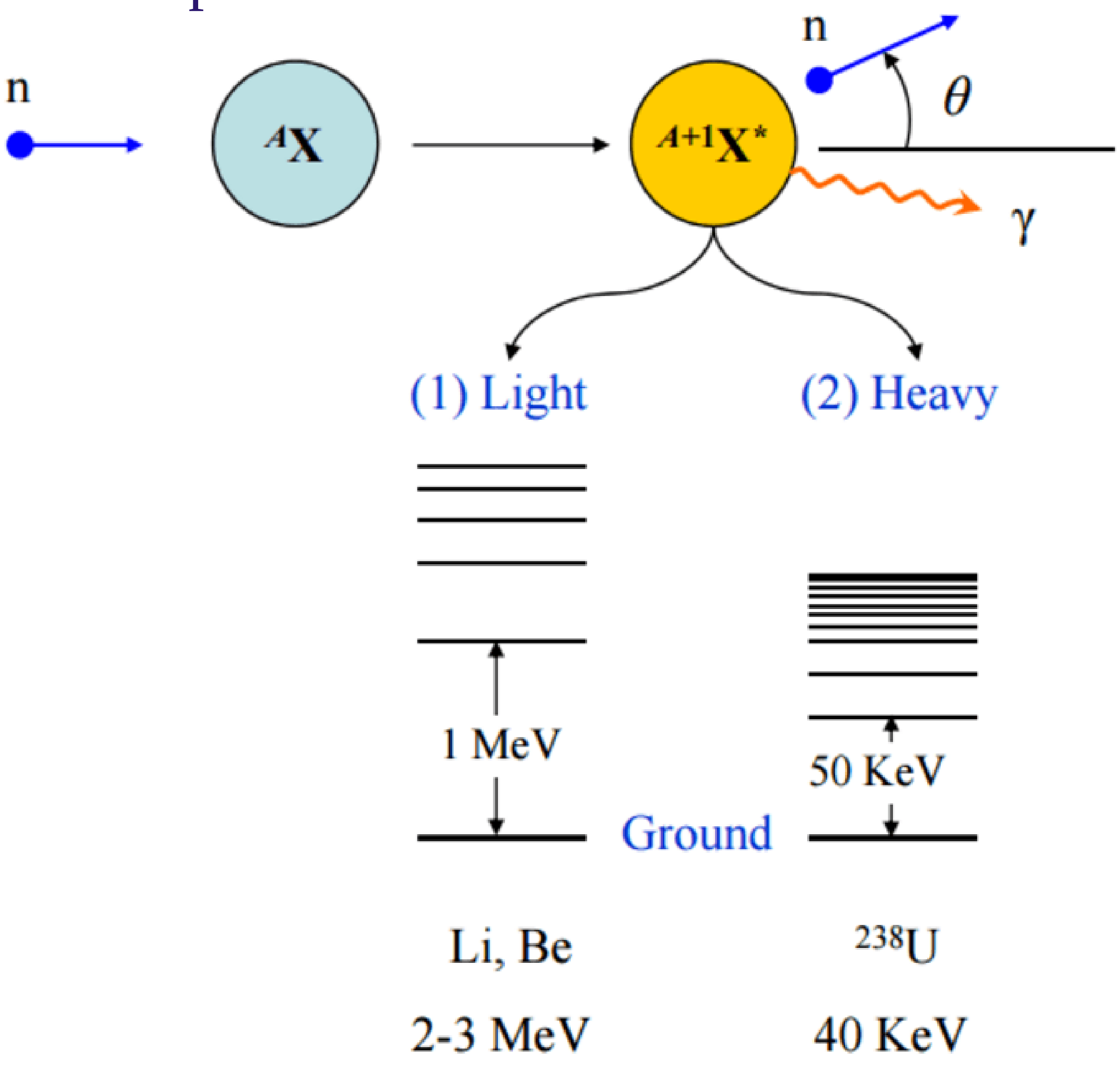
- Le Tan Phuc (DTU)

Da Nang, March 10, 2017

CONTENT

INTRODUCTION

The neutron-capture resonances



1. Nuclear level density: the number of excited levels per unit of excitation energy (ρ)

2. The average level spacing: ($\bar{D}$)

$$\bar{D} = \frac{1}{\rho}$$

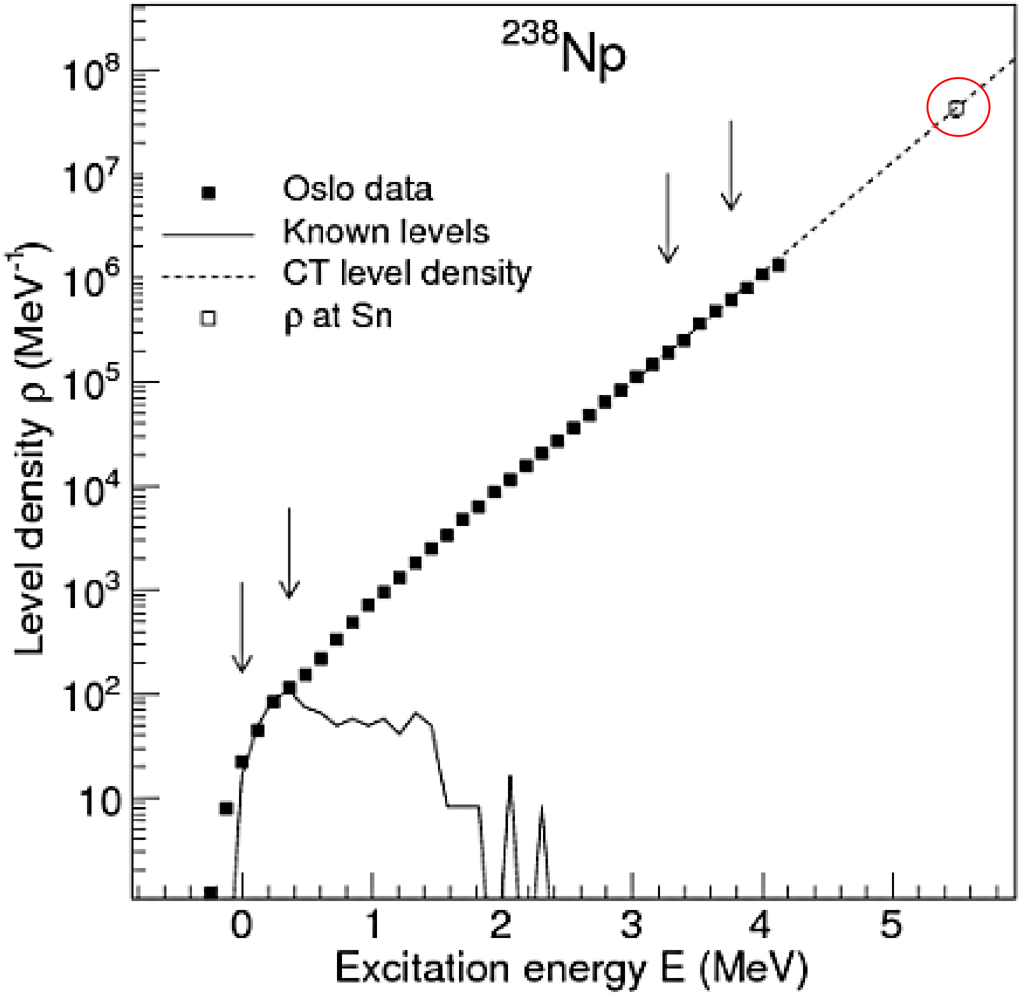
INTRODUCTION

1. Nuclear level density: the number of excited levels per unit of excitation energy

2. The average level spacing:

$$\bar{D} = \frac{1}{\rho}$$

3 . The average level spacing $\bar{D}$ at the neutron binding energy B_n



Guttormsen, M. et al, *Experimental level densities of atomic nuclei*, Eur.Phys.J. A51 (2015) no.12, 170

INTRODUCTION

The phenomenological models

The microscopic models

INTRODUCTION

The phenomenological models

- Bethe formula
- Constant Temperature Model
- Back-shifted Fermi gas Model

* Fermi gas Model

- A gas of non-interacting nucleon confined to the nuclear volume
- Hamiltonian:

$$H_0 = \sum_k \varepsilon_k (a_k^\dagger a_k)$$

$a_k^\dagger (a_k)$: creation (annihilation) operators of a single−particle with angular momentum k

- Square potential well:

$$V(r) = \begin{cases} -V_0 & for\ r \le R_0 \\ +\infty & for\ r > R_0 \end{cases}$$

$R_0 = r_0 A^{1/3}$

$r_0 \simeq 1.2 (fm)$

$V_0 \simeq 50 (MeV)$

- Single-particle spectra: having equally spaced energy levels

INTRODUCTION

The phenomenological models

1. Bethe formula of level spacing [1]

- The non-interacting Fermi gas model

$$D = 4.1 \cdot 10^6 x^4 e^{-x} / (2J+1)$$

$$x = (AE^*)^{\frac{1}{2}} / 2.20$$

J: The angular-momentum

$$\bar{D} = D_{E^* = B_n}$$

A: the mass number of the nucleus

E : the excitation energy

H.A. Bethe, Phys. Rev. 50 (1936) 332.

INTRODUCTION

The phenomenological models

2. Constant Temperature Model at low excitation energies [2]

3. The back-shifted Fermi-gas model[3]

Bethe formula + shell + pairing effects.

2. A. Gilbert and A.G.W. Cameron, Can. J. Phys. 43 (1965) 1446

3. W. Dilg, W. Schantl, H. Vonach, M. Uhl, Nucl. Phys. A 217 (1973) 269.

INTRODUCTION

The phenomenological models

2. Constant Temperature Model at low excitation energies [2]

- The angular-momentum-dependent level density at low excitation energies has form as:

$$If \qquad E^* \leq E_M :$$

$$\rho\left(E^*, J\right) = \frac{1}{T} . \exp\left(\frac{E^* - E_0}{T}\right) . \frac{2J+1}{2\sigma^2} . \exp\left\{-\frac{(J+1/2)^2}{2\sigma^2}\right\} \qquad ,$$

where

$$E_M = 2.5 + \frac{150}{A} + \Delta(N) + \Delta(Z)$$

T : the nuclear temperature, T = constant

E_0 : the energy-shift, empirical parameters

σ : the spin-cut off parameter

A. Gilbert and A.G.W. Cameron, Can. J. Phys. 43 (1965) 1446

INTRODUCTION

The phenomenological models

3. The back-shifted Fermi-gas model[3]

- The angular-momentum-dependent level density has form as:

$$\rho\left(E^{*},J\right)\sim\exp\left(2\sqrt{aU}\right)$$

$$U=E^{*}-\Delta(N,Z)$$

$$a=\frac{0.00917S(N,Z)-0.120}{A}$$

$$\Delta(N,Z)=\begin{cases}2\delta A^{-\frac{1}{2}} & \text{(doubly even)}\\ \delta A^{-\frac{1}{2}} & \text{(odd mass)}\\ \mu A^{-1} & \text{(doubly odd)}\end{cases}$$

a is the level density parameter,

S(N,Z): total shell correction

Δ(N,Z): pairing energies

S , Δ: empirical parameters

$$\delta=12.8\ MeV,\ \mu=29.4\ MeV,$$

W. Dilg, W. Schantl, H. Vonach, M. Uhl, Nucl. Phys. A 217 (1973) 269.

INTRODUCTION

The phenomenological models

The average s-wave resonance spacing can be related to the spin-dependent level density $\rho(E^{*}, J)$ as:

$$\bar{D} = \frac{10^6}{\rho\left(B_n, I_t - \frac{1}{2}\right) + \rho\left(B_n, I_t + \frac{1}{2}\right)} \qquad for\ I_t > 0,$$

$$= \frac{1}{\rho(B_n, 1/2)} \qquad for\ I_t = 0$$

I : the target spin

B_n : the neutron binding energy

INTRODUCTION

The phenomenological models

3. The back-shifted Fermi-gas model[3]

- Improve the Gilbert and Cameron approach by provide level density parameter for the back-shifted in the mass range 40 < A < 250

$$\rho\left(E^*, J\right) \sim \exp\left(2\sqrt{aU}\right)$$

$$U = E^* - \Delta(N,Z)$$

$$a = \frac{0.00917\,S(N,Z) - 0.120}{A}$$

a is the level density parameter,
S(N,Z): total shell correction
Δ(N,Z): pairing energies
S , Δ: empirical parameters

$$\Delta(N,Z) = \begin{cases} 2\delta A^{-\frac{1}{2}} & \text{(doubly even)} \\ \delta A^{-\frac{1}{2}} & \text{(odd mass)} \\ \mu A^{-1} & \text{(doubly odd)} \end{cases}$$

$$\delta = 12.8\ MeV,\ \mu = 29.4\ MeV,$$

W. Dilg, W. Schantl, H. Vonach, M. Uhl, Nucl. Phys. A 217 (1973) 269.

INTRODUCTION

The microscopic models

1. The average level spacing at the neutron binding energy B_n, the ground-state spin of the target nucleus I_t is given as

$$\bar{D} = \frac{10^6}{r\left(B_n, I_t - \frac{1}{2}\right) + r\left(B_n, I_t + \frac{1}{2}\right)} \quad \text{for } I_t > 0,$$

$$= \frac{1}{r(B_n, 1/2)} \quad \text{for } I_t = 0$$

Maino's approach[4]

Statistical thermodynamic theories: Bardeen-Cooper-Schrieffer (BCS) theory at finite temperature [2]

Goriely's approach[5]

Hartree-Fock-Bogoliubov plus combinatorial method

4. G. Maino et. al, Nuovo Cimento A 50 (1979) 1; G. Maino et. al, Nuovo Cimento A 57 (1980) 427; V. Benzi, G. Maino, and E. Menapace, Nuovo Cimento A 66 (1981) 1

5. S. Goriely, Nucl. Phys. A 605 (1996) 28; S. Goriely, S. Hilaire, and A. J. Koning, Phys. Rev. C 78 (2008) 064307 ; S. Hilaire and S. Goriely, Nucl. Phys. A 779 (2006) 63.

INTRODUCTION

Maino's approach[4]

1. Pairing Hamiltonian:

$$H = \sum_k \varepsilon_k (a_k^\dagger a_k + a_{-k}^\dagger a_{-k}) - G \sum_{k,k'} a_k^\dagger a_{-k}^\dagger a_{-k'} a_{k'}$$

$a_{\pm k}^\dagger (a_{\pm k})$: creation (annihilation) operators of a single−particle with angular momentum k

G : the constant indicating the strength of the pairing interaction

2. FTBCS equations:

$$\frac{2}{G} = \sum_{k=k_1}^{k_2} \frac{1}{E_k} tgh \frac{\beta E_k}{2},$$

$$N = \sum_{k=k_1}^{k_2} \left[1 - \frac{\varepsilon_k - \lambda}{E_k} tgh \frac{\beta E_k}{2} \right],$$

$$E = \sum_{k=k_1}^{k_2} \varepsilon_k \left[1 - \frac{\varepsilon_k - \lambda}{E_k} tgh \frac{\beta E_k}{2} \right] - \frac{\Delta^2}{G}$$

$k_1 = 1$, $k_2 = 2k_F$

k_F : the last filled level at zero temperature

4. G. Maino et. al, Nuovo Cimento A 50 (1979) 1; G. Maino et. al, Nuovo Cimento A 57 (1980) 427; V. Benzi, G. Maino, and E. Menapace, Nuovo Cimento A 66 (1981) 1

INTRODUCTION

Maino’s approach

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$$E = \sum_{k=k_1}^{k_2} \varepsilon_k \left[1 - \frac{\varepsilon_k - \lambda}{E_k} tgh \frac{\beta E_k}{2} \right] - \frac{\Delta^2}{G}$$

$\Delta(T)$ pairing gap

$\mathrm{E}(T)$ total energy

$\mathrm{S}(T)$ entropy

$\mathrm{C}(T)$ heat capacity

$\omega(\mathrm{E}^*)$ total state density

FORMALISM

Maino's approach

3. By assuming that distribution of the total spin of the nucleus is given in terms of the Gaussian distribution, the angular-momentum-dependent level density can be approximately calculated via the total state density as

$$\rho\left(E^{*},J\right)\approx\frac{1}{\sqrt{8\pi\sigma_{\|}^{2}(E^{*})}}\omega(\mathrm{E}^{*})\sum_{K=-J}^{+J}\left[-\frac{K^{2}}{2\sigma_{\|}^{2}(E^{*})}-\frac{J(J+1)-\mathrm{K}^{2}}{2\sigma_{\perp}^{2}(E^{*})}\right]$$

K: the projection of total angular momentum J on the symmetry axis

$\sigma_{\|}$: the parallel spin cut-off parameter

$\sigma_{\perp}$: the perpendicular spin cut-off parameter

FORMALISM

Goriely's approach[5]

1. Solving the Hartree–Fock–Bogoliubov equation (pairing is taken into account) with the use of effective nucleon-nucleon (NN) interaction of the Skyrme type [5] → find ground state and neutron (proton) single-particle spectra.

- Single particle level scheme for protons:

$$\varepsilon_1^{\pi} < \varepsilon_2^{\pi} < \ldots < \varepsilon_n^{\pi} < \ldots$$

- Single particle level scheme for neutrons:

$$\varepsilon_1^{\nu} < \varepsilon_2^{\nu} < \ldots < \varepsilon_n^{\nu} < \ldots$$

5. S. Goriely, Nucl. Phys. A 605 (1996) 28; S. Goriely, S. Hilaire, and A. J. Koning, Phys. Rev. C 78 (2008) 064307 ; S. Hilaire and S. Goriely, Nucl. Phys. A 779 (2006) 63.

FORMALISM

Goriely's approach[5]

2. The single particle–single hole states are defined as

- for proton particles:

$$\left.\begin{array}{l} \varepsilon_i^1 = \varepsilon_{Z+i}^{\pi} - \varepsilon_Z^{\pi} \\ m_i^1 = 2m_{Z+i}^{\pi} \\ p_i^1 = p_{Z+i}^{\pi} \\ \Delta_i^1 = \Delta_{Z+i}^{\pi} \end{array}\right\}, \quad i = 1, \ldots, I_1,$$

- for proton holes:

$$\left.\begin{array}{l} \varepsilon_i^2 = \varepsilon_Z^{\pi} - \varepsilon_{Z-i+1}^{\pi} \\ m_i^2 = -2m_{Z-i+1}^{\pi} \\ p_i^2 = p_{Z-i+1}^{\pi} \\ \Delta_i^2 = \Delta_{Z-i+1}^{\pi} \end{array}\right\}, \quad i = 1, \ldots, I_2,$$

- for neutron particles:

$$\left.\begin{array}{l} \varepsilon_i^3 = \varepsilon_{N+i}^{\nu} - \varepsilon_N^{\nu} \\ m_i^3 = 2m_{N+i}^{\nu} \\ p_i^3 = p_{N+i}^{\nu} \\ \Delta_i^3 = \Delta_{N+i}^{\pi} \end{array}\right\}, \quad i = 1, \ldots, I_3,$$

- for neutron holes:

$$\left.\begin{array}{l} \varepsilon_i^4 = \varepsilon_N^{\nu} - \varepsilon_{N-i+1}^{\nu} \\ m_i^4 = -2m_{N-i+1}^{\nu} \\ p_i^4 = p_{N-i+1}^{\nu} \\ \Delta_i^4 = \Delta_{N-i+1}^{\pi} \end{array}\right\}, \quad i = 1, \ldots, I_4,$$

FORMALISM

Goriely's approach[5]

2. The single particle–single hole states are defined as

- for proton particles:

$$\Delta_i^1 = \Delta_{Z+i}^{\pi}$$

- for proton holes:

$$\Delta_i^2 = \Delta_{Z-i+1}^{\pi}$$

- for neutron particles:

$$\Delta_i^3 = \Delta_{N+i}^{\pi}$$

- for neutron holes:

$$\Delta_i^4 = \Delta_{N-i+1}^{\pi}$$

$$\Delta(Z, N)$$

FORMALISM

Goriely's approach[5]

3. The state density ω (E*, M, P) for a given excitation energy E*, , a given spin projection M and a given parity P is the number of solutions per unit energy of the set of equations

$$E^* = \sum_{k=1}^{4}\sum_{i=1}^{I_k} n_i^k \varepsilon_i^k$$

$$M = \sum_{k=1}^{4}\sum_{i=1}^{I_k} n_i^k m_i^k ,$$

$$P = \prod_{k=1}^{4}\prod_{i=1}^{I_k} (p_i^k)^{n_i^k} ,$$

$$N_k = \sum_{i=1}^{I_k} n_i^k ,$$

I_k : denote the number of discrete states considered for each sets of single-particle–single-hole states

k = 1, 2, 3, or 4

$n_i^k = 0$ or 1 if the state *i* is empty or occupied

$$\omega(\mathrm{E}^*, M, P)$$

5. S. Goriely, Nucl. Phys. A 605 (1996) 28; S. Goriely, S. Hilaire, and A. J. Koning, Phys. Rev. C 78 (2008) 064307 ; S. Hilaire and S. Goriely, Nucl. Phys. A 779 (2006) 63.

FORMALISM

Goriely's approach[5]

4. Angular-momentum-dependent NLD

$$\rho\left(E^*, J, P\right) = \omega\left(E^*, M = J, P\right) - \omega\left(E^*, M = J+1, P\right)$$

exact formula of the *J*-dependent NLD

5. S. Goriely, Nucl. Phys. A 605 (1996) 28; S. Goriely, S. Hilaire, and A. J. Koning, Phys. Rev. C 78 (2008) 064307 ; S. Hilaire and S. Goriely, Nucl. Phys. A 779 (2006) 63.

METHOD

FTABCS

1. BCS theory at finite temperature and finite angular momentum (FTABCS) [6]:

$$H = \sum_k \varepsilon_k (a_k^\dagger a_k + a_{-k}^\dagger a_{-k}) - G \sum_{k,k'} a_k^\dagger a_{-k}^\dagger a_{-k'} a_{k'} - \lambda \hat{N} - \gamma \hat{M}$$

$a_{\pm k}^\dagger (a_{\pm k})$: creation (annihilation) operators of a single–particle with angular momentum k

ε_k : the single-particle energy level

λ : the chemical potential

γ : rotational or angular velocity

N : the particle–number operator

M : the total angular momentum operator

G : the constant indicating the strength of the pairing interaction

$$N = \sum_k \left(a_k^\dagger a_k + a_{-k}^\dagger a_{-k} \right),$$

$$M = \sum_k m_k \left(a_k^\dagger a_k - a_{-k}^\dagger a_{-k} \right)$$

m_k : the single-particle spin projections

6. L. G. Moretto, Nucl. Phys. A 185 (1972) 145

METHOD

FTABCS

2. The FTABCS equations for the pairing gap Δ, particle number *N*, and projection of total angular momentum *M* [7]

$$\Delta = G\sum_k u_k v_k \left(1 - n_k^+ - n_k^-\right),$$

$$N = 2\sum_k \left[(1 - n_k^+ - n_k^-)v_k^2 + \frac{1}{2}(n_k^+ + n_k^-) \right]$$

$$M = \sum_k m_k (n_k^+ - n_k^-),$$

$$n_k^{\pm} = \frac{1}{1 + e^{\beta(E_k \mp \gamma m_k)}}$$

$\Delta(T,M)$	pairing gap
$E(T,M)$	total energy
$S(T,M)$	entropy
$C(T,M)$	heat capacity
$\omega(E^*,M)$	total state density

7. N. Quang Hung and N. Dinh Dang, Phys. Rev. C 78 (2008) 064315 .

METHOD

FTABCS

3. Angular-momentum-dependent NLD

$$\rho\left(E^*, J\right) = \omega\left(E^*, M = J\right) - \omega\left(E^*, M = J + 1\right)$$

exact formula of the *J*-dependent NLD

4. The average level spacing at the neutron binding energy B_n, the ground-state spin of the target nucleus I_t is given as

$$\bar{D} = \frac{10^6}{r\left(E^* = B_n, J = I_t - \frac{1}{2}\right) + r\left(E^* = B_n, J = I_t + \frac{1}{2}\right)}$$

METHOD

1. G. Maino [1]:

+ BCS theory at finite temperature

+ Nilsson potential

+ $\Delta(Z, N, T)$

+ $\Delta_N(T=0) = \Delta_Z(T=0)$

+ $\omega_{intr}(E^*)$

+ approximate formula of the J-dependent NLD

2. S. Goriely [2]:

+ Hartree-Fock-Bogoliubov plus combinatorial method

+ $\Delta(Z, N)$

+ $\omega_{intr}(E^*, M, P)$

+ exact formula of the J-dependent NLD

+ NLD is obtained by fitting to the experimental NLD

3.FTABCS [3]:

+ BCS theory at finite temperature and finite angular momentum

+ Woods-Saxon (WS) potential with a global parameter set

+ $\Delta(Z, N, T, M)$

+ $W_{intr}(E^*, M)$

+ exact formula of the J-dependent NLD

RESULTS

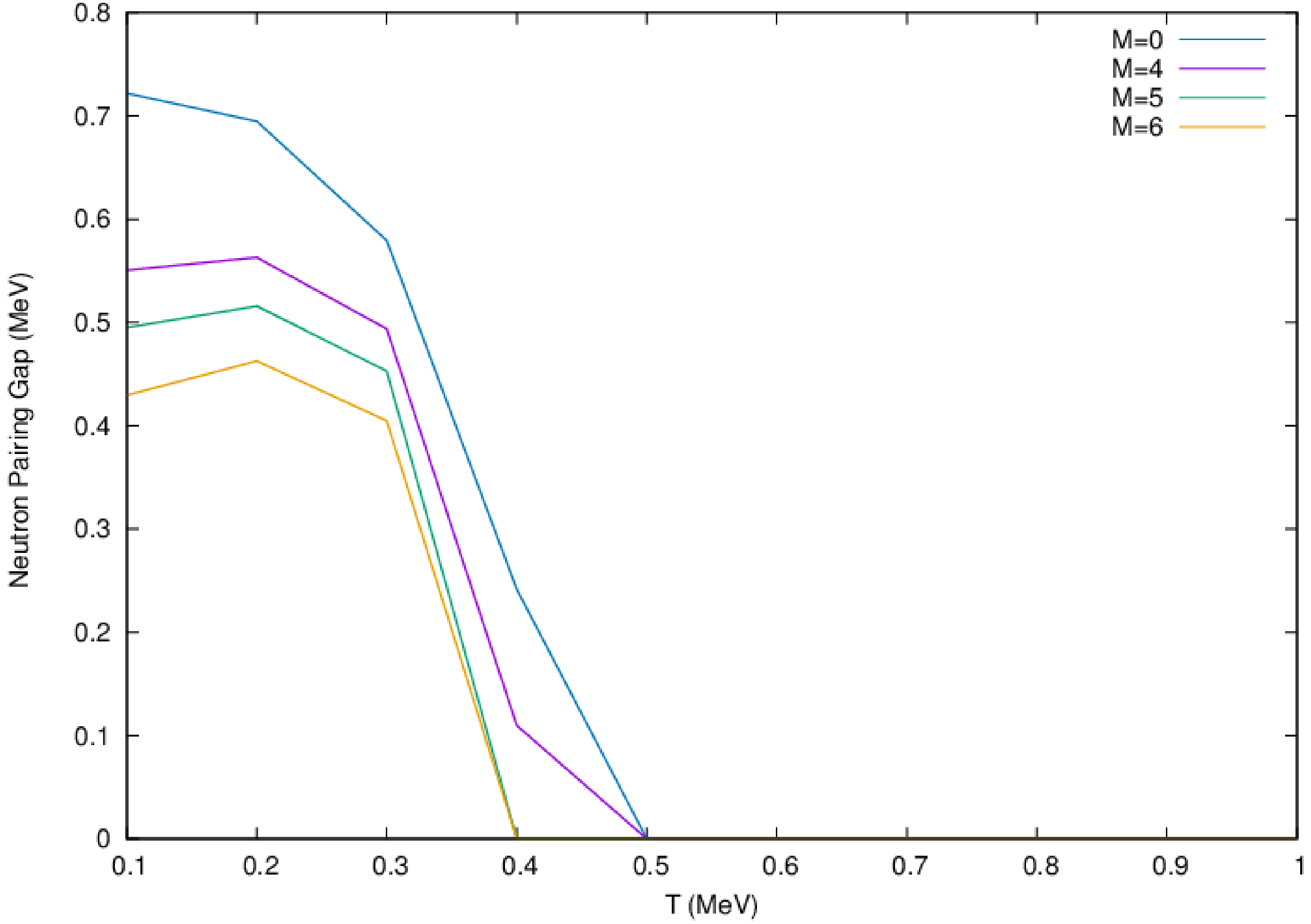
Ingredient of numerical calculation

- Single-particle spectra: the axially deformed Woods-Saxon with the quadrupole β_2 and hexadecapole β_4 deformation parameters.

- Pairing interaction parameters G_N and G_Z

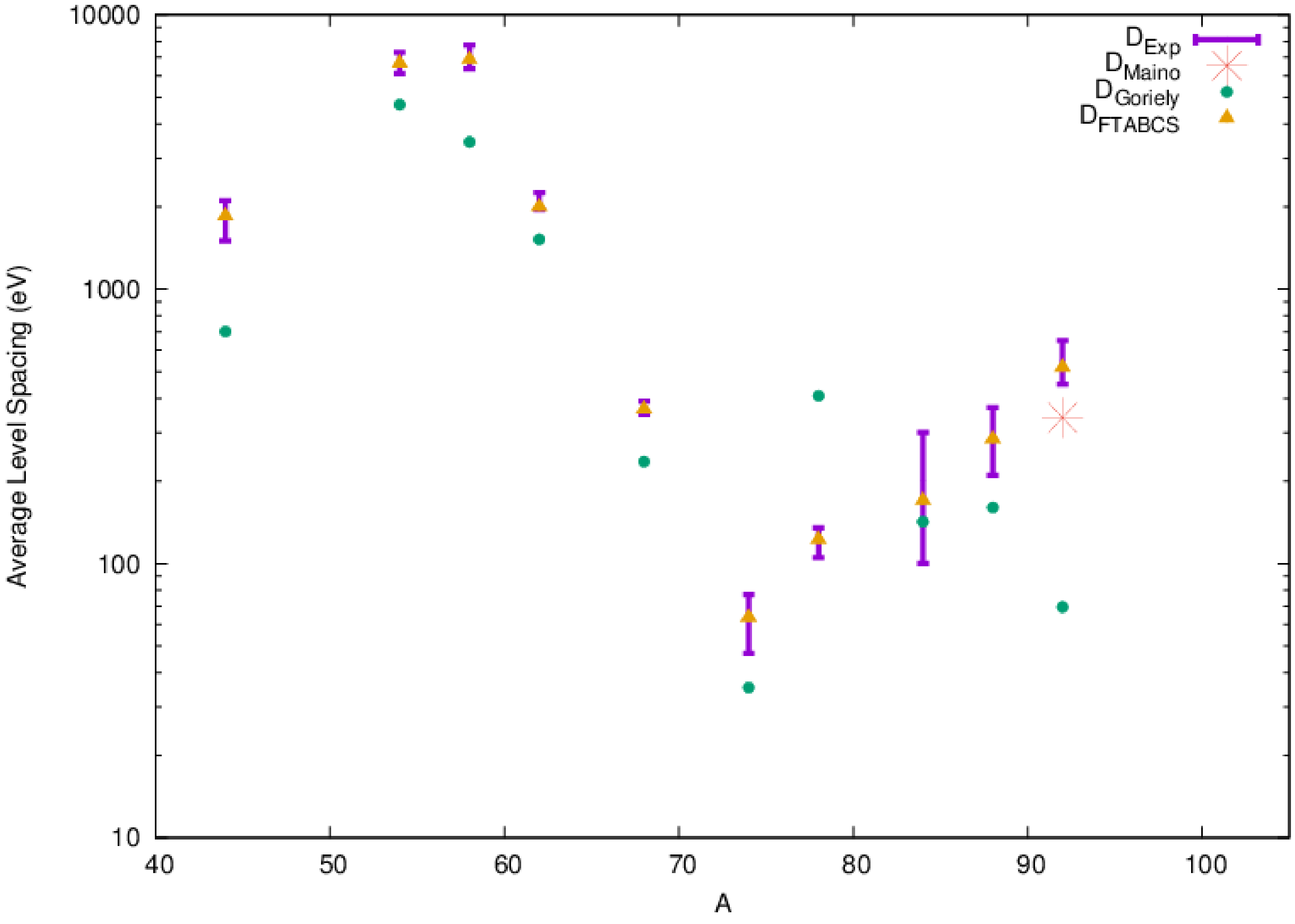
$$\Delta_N = 11.56N^{-0.552}, \Delta_Z = 11.4Z^{-0.567}.$$

FTABCS: ^{250}Cf (I = 9/2)



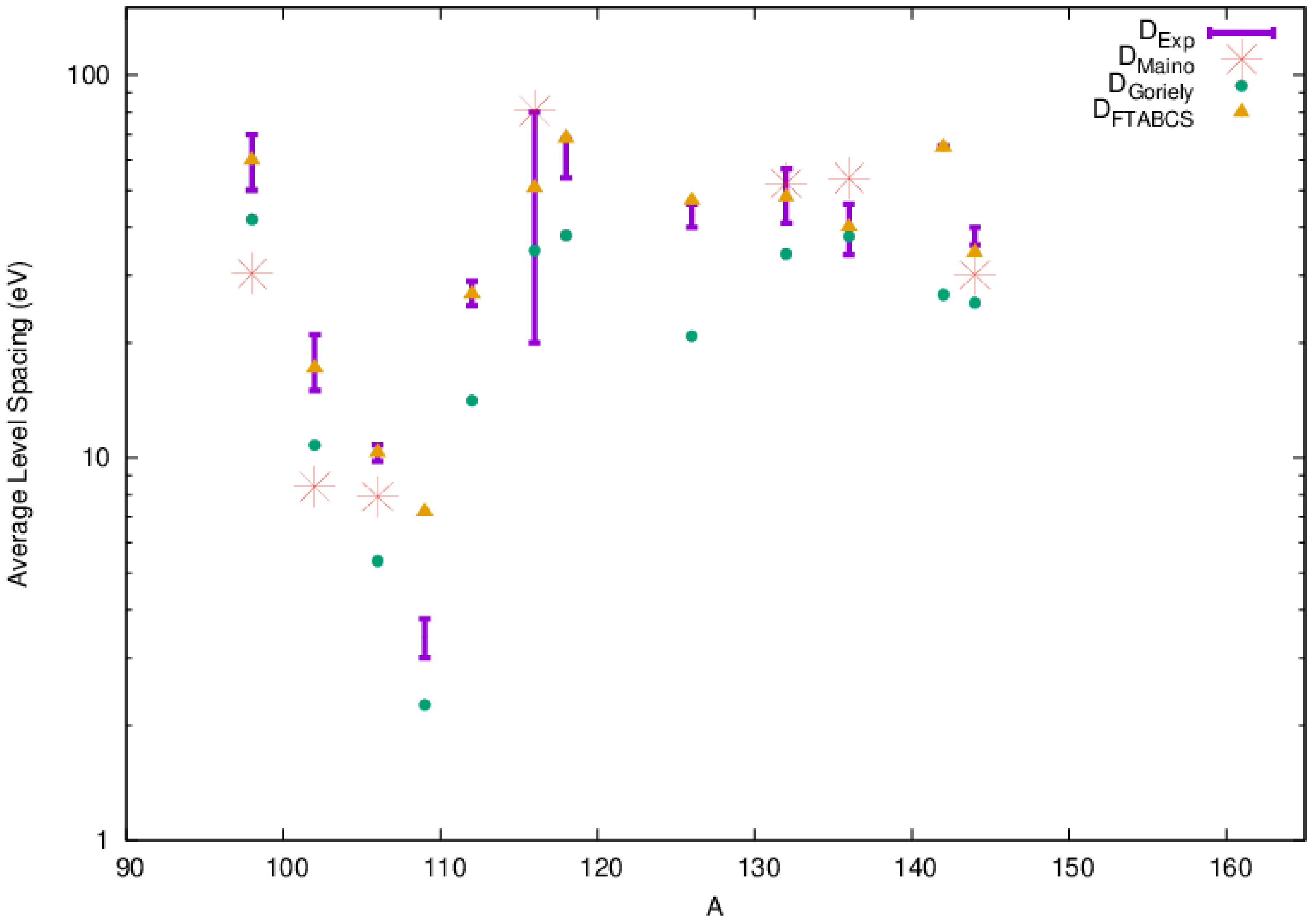
RESULTS

$40 \leq A \leq 95$



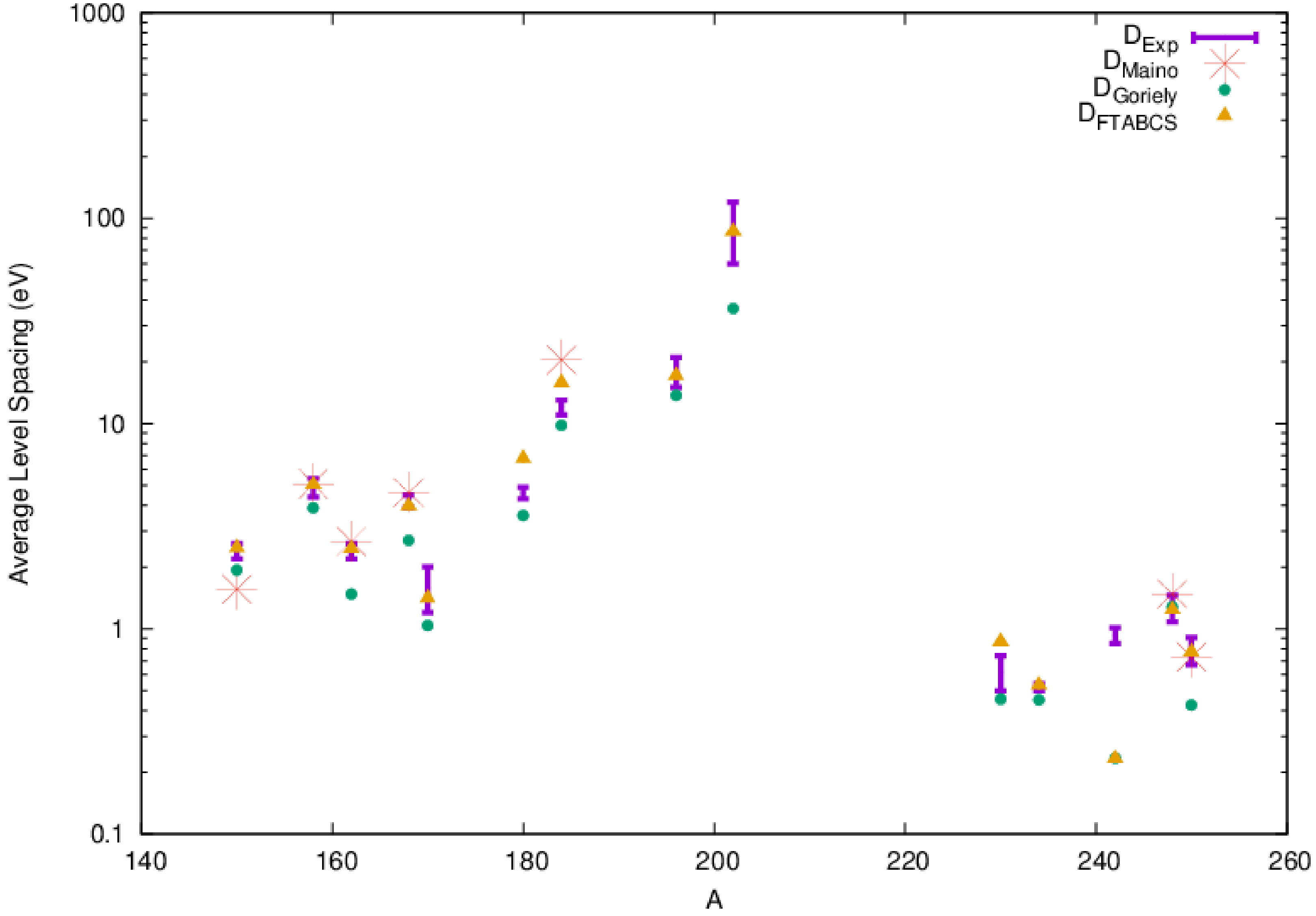
RESULTS

$$96 \leq A \leq 145$$



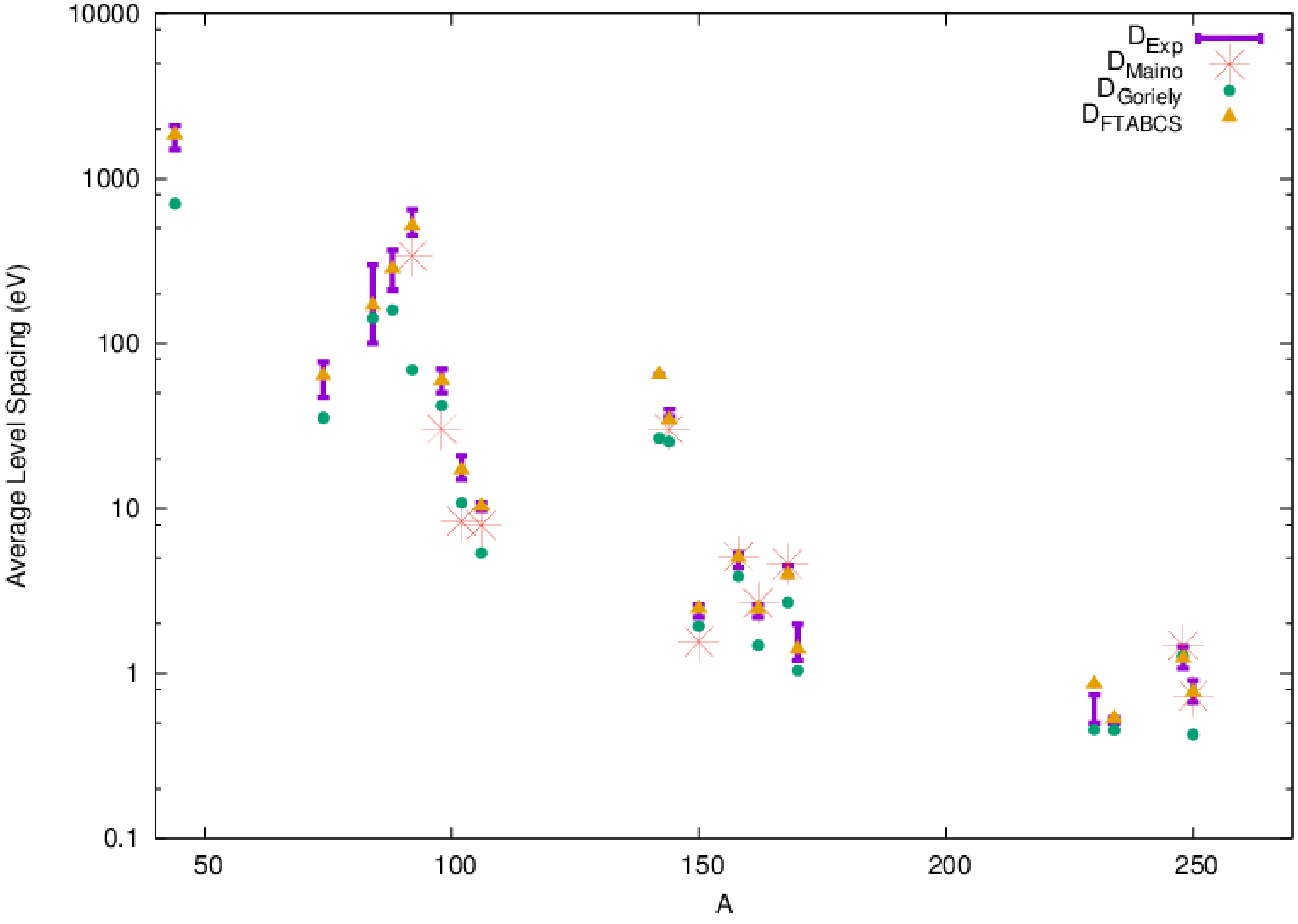
RESULTS

$$145 \le A \le 250$$



RESULTS

$$I_t = \begin{cases} \frac{5}{2} \\ \frac{7}{2} \\ \frac{9}{2} \end{cases}$$



CONCLUSIONS

- The average level spacings are calculated within the FTABCS for several even-even nuclei from the medium ^{44}Ca to heavy ^{250}Cf mass isotopes
- The results obtained show that the average level spacing obtained within our FTABCS are in better agreement with the experimental data than those proposed by Maino and Goriely, especially for nucleus with large ground-state spin whose J-dependent pairing gaps and NLD are important in calculating *D*.

CONCLUSIONS

- The merit of present approach is that it is rather simple with only two parameters G_N and G_Z, which are adjusted to reproduce the experimental pairing gaps at $T=0$ and $M=0$.
- The results are restricted to even-even nuclei

CONCLUSIONS

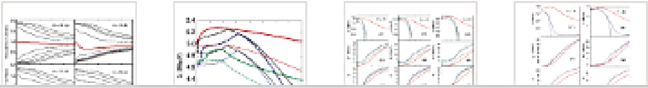
PHYSICAL REVIEW C
covering nuclear physics

Improved treatment of blocking effect at finite temperature

N. Quang Hung, N. Dinh Dang, and L. T. Quynh Huong
Phys. Rev. C **94**, 024341 – Published 30 August 2016

ABSTRACT

The blocking effect caused by the odd particle on the pairing properties of systems with an odd number of fermions at finite temperature interacting via the monopole pairing force is studied within several approximations. The results are compared with the predictions obtained by using the exact solutions of the pairing Hamiltonian. The comparison favors the approximation with the odd particle occupying the top level, which is the closest to the Fermi surface and whose occupation number decreases with increasing temperature.



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