QCD phase diagram in the vector meson extended PQM model

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Based on: P. Kovács, Zs. Szép, Gy. Wolf, PRD93 (2016) 114014
Overview

1. Introduction
   - Motivation

2. The model
   - Axial(vector) meson extended linear $\sigma$-model
   - Parametrization at $T = 0$
   - Polyakov loop

3. eLSM at finite $T/\mu_B$
   - Extremum equations for $\phi_{N/S}$ and $\Phi, \bar{\Phi}$

4. Results
   - $T$ dependence of the order parameters
   - Critical endpoint
   - Phase diagram

5. Summary
QCD phase diagram

Phase diagram in the $T - \mu_B - \mu_I$ space

- At $\mu_B = 0$ $T_c = 151$ MeV
- Is there a CP?
  ($T_{CP} = 162$ MeV, $\mu_{CP} = 360$ MeV, Fodor-Katz)
- At $T = 0$ in $\mu_B$ where is the phase boundary?

Details of the phase diagram are heavily studied theoretically (Lattice, EFT), and experimentally (RHIC, LHC, FAIR, NICA)
Motivation

- At $\mu = 0$ we know the properties of strong interactions from the lattice in theoretical side and from STAR/PHENIX and from ALICE in the experimental side. On the other hand, for $\mu \gg 0$ at the moment no theory and no experiment provide reasonable information.
- What is the order of phase transition on the $T=0$ line? Is there a CEP?
- Equation of state for neutron stars.
- How the masses change in medium?

Idea

- Build an effective model having the right global symmetry pattern.
- Compare the thermodynamics of the model with lattice at $\mu = 0$.
- Extrapolate to high $\mu$. 
Effective models

Since QCD is very hard to solve $\rightarrow$ low energy effective models were set up $\rightarrow$ reflecting the global symmetries of QCD

- Nambu-Jona-Lasinio model (+Kobayashi-Maskawa-t’Hooft)
- Chiral perturbation theory
- Linear and nonlinear (it does not contain degrees of freedom relevant at high T) sigma model
- To study the phase diagram, we introduced the constituent quarks
- For mimicking confinement, we add the Polyakov loops.

extended Polyakov-Quark-Meson model

Similar model e.g.: Pisarski, Skokov, Phys.Rev. D94 (2016) 034015
Chiral symmetry

If the quark masses are zero (chiral limit) \( \Rightarrow \) QCD invariant under the following global transformation (chiral symmetry):
\[
q_L = (1 - \gamma_5)/2q, \quad q_R = (1 + \gamma_5)/2q
\]
only the mass term mixes
\[
U(3)_V q = \exp(-i\alpha t)q \quad U(3)_A q = \exp(-i\beta\gamma_5 t)q
\]

\[
U(3)_L \times U(3)_R \simeq U(3)_V \times U(3)_A =
SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A
\]

by any quark mass \( SU(3)_V \times U(1)_V \times U(1)_A \) remains
\[
U(1)_V \text{ term } \longrightarrow \text{ baryon number conservation}
\]
\[
U(1)_A \text{ term } \longrightarrow \text{ broken through axial anomaly}
\]
\[
SU(3)_V \text{ term } \longrightarrow \text{ broken down to } SU(2)_V \text{ if } m_u = m_d \neq m_s
\]
\[
\longrightarrow \text{ totally broken if } m_u \neq m_d \neq m_s \text{ (in nature)}
\]
Meson fields - pseudoscalar and scalar meson nonets

\[ \Phi_{PS} = 8 \sum_{i=0}^{8} \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_{N+0}^0 \sqrt{2} \\ \pi^- \\ K^- \end{pmatrix} \begin{pmatrix} \eta_{N-0}^0 \sqrt{2} \\ \pi^+ \\ K^+ \end{pmatrix} \begin{pmatrix} \pi^- \\ \eta_{N-0}^0 \sqrt{2} \\ K^0 \end{pmatrix} \begin{pmatrix} K^- \\ \eta_S \end{pmatrix} \sim \bar{q}_i \gamma_5 q_j \]

\[ \Phi_{S} = \sum_{i=0}^{8} \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \sigma_{N+a_0^0} \sqrt{2} \\ a_0^- \\ K_S^- \end{pmatrix} \begin{pmatrix} a_0^+ \\ \sigma_{N-a_0^0} \sqrt{2} \\ K_S^0 \end{pmatrix} \begin{pmatrix} K_S^+ \\ \sigma_S \end{pmatrix} \sim \bar{q}_i q_j \]

Particle content:
Pseudoscalars: \( \pi(138), K(495), \eta(548), \eta'(958) \)
Scalars: \( a_0(980 \text{ or } 1450), K_0^*(800 \text{ or } 1430), \)
(\( \sigma_N, \sigma_S \) : 2 of \( f_0(500, 980, 1370, 1500, 1710) \))
### Structure of scalar mesons

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<td>$A_0(980)$</td>
<td>980 ± 20</td>
<td>50 – 100</td>
<td>$\pi\pi$ dominant</td>
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<tr>
<td>$A_0(1450)$</td>
<td>1474 ± 19</td>
<td>265 ± 13</td>
<td>$\pi\eta, \pi\eta', K\bar{K}$</td>
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<td>$K_s(800) = \kappa$</td>
<td>682 ± 29</td>
<td>547 ± 24</td>
<td>$K\pi$</td>
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<td>$K_s(1430)$</td>
<td>1425 ± 50</td>
<td>270 ± 80</td>
<td>$K\pi$ dominant</td>
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<tr>
<td>$f_0(500) = \sigma$</td>
<td>400–550</td>
<td>400 – 700</td>
<td>$\pi\pi$ dominant</td>
</tr>
<tr>
<td>$f_0(980)$</td>
<td>980 ± 20</td>
<td>40 – 100</td>
<td>$\pi\pi$ dominant</td>
</tr>
<tr>
<td>$f_0(1370)$</td>
<td>1200–1500</td>
<td>200 – 500</td>
<td>$\pi\pi \approx 250, K\bar{K} \approx 150$</td>
</tr>
<tr>
<td>$f_0(1500)$</td>
<td>1505 ± 6</td>
<td>109 ± 7</td>
<td>$\pi\pi \approx 38, K\bar{K} \approx 9.4$</td>
</tr>
<tr>
<td>$f_0(1710)$</td>
<td>1722 ± 6</td>
<td>135 ± 7</td>
<td>$\pi\pi \approx 30, K\bar{K} \approx 71$</td>
</tr>
</tbody>
</table>

Possible scalar states: $\bar{q}q$, $\bar{q}\bar{q}qq$, meson-meson molecules, glueballs
pseudoscalar nonet: $\pi, K, \eta, \eta'$, scalar nonet: $A_0, K_0, 2f_0$
multiquark states: $f_0(980), A_0(980)$, $f_0(600), K_0(800)$
meson-meson bound state ($K\bar{K}$): $f_0(980)$
 glueballs: $f_0(1500)$ (weak coupling to $\gamma\gamma$), $f_0(1710)$
Included fields - vector meson nonets

\[ V^\mu = \sum_{i=0}^{8} \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega_N - \rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \sqrt{2} K^{*0} & \omega_S \end{pmatrix}^{\mu} \]

\[ A^\mu = \sum_{i=0}^{8} b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N} + a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N} - a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \sqrt{2} K_1^0 & f_{1S} \end{pmatrix}^{\mu} \]

Particle content:
Vector mesons: \( \rho(770), K^*(894), \omega_N = \omega(782), \omega_S = \phi(1020) \)
Axial vectors: \( a_1(1230), K_1(1270), f_{1N}(1280), f_{1S}(1426) \)
Axial(vector) meson extended linear $\sigma$-model

Lagrangian (2/1)

\[
\mathcal{L}_{\text{Tot}} = \text{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 \text{Tr}(\Phi^\dagger \Phi)^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 \\
- \frac{1}{4} \text{Tr}(L^2_{\mu\nu} + R^2_{\mu\nu}) + \text{Tr} \left[ \left( \frac{m_1^2}{2} + \Delta \right) (L^2_\mu + R^2_\mu) \right] + \text{Tr}[H(\Phi + \Phi^\dagger)] \\
+ c_1 (\det \Phi + \det \Phi^\dagger) + i \frac{g_2}{2} (\text{Tr}\{L_{\mu\nu} [L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu} [R^\mu, R^\nu]\}) \\
+ \frac{h_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(L^2_\mu + R^2_\mu) + h_2 \text{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \text{Tr}(L_\mu \Phi R^\mu \Phi^\dagger). \\
+ \bar{\psi} i \not{\partial} \psi - g_F \bar{\psi} (\Phi_S + i \gamma_5 \Phi_{PS}) \psi + g_V \bar{\psi} \gamma^\mu \left( V_\mu + \frac{g_A}{g_V} \gamma_5 A_\mu \right) \psi \\
+ \text{Polyakov loops}
\]

where

\[ D^\mu \Phi = \partial^\mu \Phi - ig_1 (L^\mu \Phi - \Phi R^\mu) - ieA_e^{\mu} [T_3, \Phi] \]

\[ \Phi = \sum_{i=0}^{8} (\sigma_i + i\pi_i) T_i, \quad H = \sum_{i=0}^{8} h_i T_i \quad T_i : U(3) \text{ generators} \]

\[ R^\mu = \sum_{i=0}^{8} (\rho_i^\mu - b_i^\mu) T_i, \quad L^\mu = \sum_{i=0}^{8} (\rho_i^\mu + b_i^\mu) T_i \]

\[ L^{\mu\nu} = \partial^\mu L^\nu - ieA_e^{\mu} [T_3, L^\nu] - \{ \partial^\nu L^\mu - ieA_e^{\nu} [T_3, L^\mu] \} \]

\[ R^{\mu\nu} = \partial^\mu R^\nu - ieA_e^{\mu} [T_3, R^\nu] - \{ \partial^\nu R^\mu - ieA_e^{\nu} [T_3, R^\mu] \} \]

\[ \bar{\Psi} = (\bar{u}, \bar{d}, \bar{s}) \]

non strange – strange base:

\[ \varphi_N = \sqrt{2/3} \varphi_0 + \sqrt{1/3} \varphi_8, \]

\[ \varphi_S = \sqrt{1/3} \varphi_0 - \sqrt{2/3} \varphi_8, \quad \varphi \in (\sigma_i, \pi_i, \rho_i^\mu, b_i^\mu, h_i) \]

broken symmetry: non-zero condensates \( \langle \sigma_N \rangle, \langle \sigma_S \rangle \mapsto \phi_N, \phi_S \)
Symmetry properties of the model

Global $U(3)_L \times U(3)_R$ transformation:

$$\Phi \rightarrow U_L \Phi U_R^\dagger$$

$$L^\mu \rightarrow U_L L^\mu U_L^\dagger \quad R^\mu \rightarrow U_R R^\mu U_R^\dagger$$

Consequences (using the unitarity of U’s):

$$D^\mu \Phi \rightarrow U_L D^\mu \Phi U_R^\dagger$$

$$L^{\mu \nu} \rightarrow U_L L^{\mu \nu} U_L^\dagger \quad R^{\mu \nu} \rightarrow U_R R^{\mu \nu} U_R^\dagger$$

All terms are invariant except

the determinant: breaks $U_A(1)$

the explicit symmetry breaking term H: breaks $SU_A(3)$ and remains $U_V(1) \times SU_V(3)$ if all 3 eigenvalues of H are equal

remains $U_V(1) \times SU_V(2)$ if 2 eigenvalues of H are equal

remains $U_V(1)$ if all 3 eigenvalues of H are different.
### Determinant term

\[
U_L = e^{-i\omega_L^a T^a} \quad U_R = e^{-i\omega_R^a T^a}
\]
\[
\omega_V^a = 0.5(\omega_L^a + \omega_R^a) \quad \omega_A^a = 0.5(\omega_L^a - \omega_R^a)
\]

By SU(3)$_L \times$SU(3)$_R$ transformation (if \(\omega_0^L = \omega_0^R = 0 = \omega_V^0 = \omega_A^0\))

\[
(d\Phi)' = \det(U_L \Phi U_R^\dagger) = \det U_L \det \Phi \det U_R^\dagger = \det \Phi
\]

Similarly \(d\Phi^\dagger\) is also invariant.

If \(\omega_V^0 \neq 0\) and all the other \(\omega\)'s are 0 (\([T^a, T^0] = 0\))

\[
(d\Phi)' = \det(e^{-i\omega_V^0 T_0} \Phi e^{i\omega_V^0 T_0}) = \det(e^{-i\omega_V^0 T_0} e^{i\omega_V^0 T_0} \Phi) = \det \Phi
\]

On the other hand, if \(\omega_A^0 \neq 0\) and all the other \(\omega\)'s are 0

\[
(d\Phi)' = \det(e^{-i\omega_A^0 T_0} \Phi e^{-i\omega_A^0 T_0}) = \det(e^{-i\omega_A^0 T_0} e^{-i\omega_A^0 T_0} \Phi) = e^{-i2\omega_A^0} \det \Phi \text{Tr} T^0
\]

So the determinant term is invariant under U(3)$_V \times$SU(3)$_A$ transformation and breaks explicitly the U(1)$_A$ symmetry.
Explicit breaking term: $\text{Tr}[\hat{\epsilon}(\Phi + \Phi^\dagger)]$

\[
\hat{\epsilon} = \sum_{i=0}^{8} \epsilon_i T_i = \begin{pmatrix}
\frac{\epsilon N}{2} & 0 & 0 \\
0 & \frac{\epsilon N}{2} & 0 \\
0 & 0 & \frac{\epsilon_S}{\sqrt{2}}
\end{pmatrix}
\]

only $\epsilon^0, \epsilon^8 \neq 0$

- axial transformation: if at least $\epsilon^0 \neq 0$ $U(3)_A$ is broken:
  \[
  (\text{Tr}[\hat{\epsilon}(\Phi)])' = \text{Tr}(e^{-i2\omega^a}_A T^a \hat{\epsilon}\Phi)
  \]

- vector transformation
  \[
  (\text{Tr}[\hat{\epsilon}(\Phi)])' = \text{Tr}(e^{-i\omega^a}_V T^a \hat{\epsilon}e^{i\omega^a}_V T^a \Phi)
  \]

Since $[\hat{\epsilon}, T^0] = 0$, $U(1)_V$ symmetry is preserved.
If all $\epsilon^a = 0$ except $\epsilon^0$, $U(3)_V$ is preserved.
If $\epsilon^8$ also non zero, then since $[T^K, T^8] = 0$ if $k = 1, 2, 3$, $U(1)_V \times SU(2)_V$ survives (isospin symmetry)
(If $\epsilon^3 \neq 0$ too, then the isospin symmetry is broken, only $U(1)_V$.)
Spontaneous symmetry breaking

Interaction is approximately chiral symmetric, spectra not, so SSB:

\[ \sigma_{N/S} \rightarrow \sigma_{N/S} + \phi_{N/S} \quad \phi_{N/S} \equiv \langle \sigma_{N/S} \rangle \]

For tree level masses we have to select all terms quadratic in the new fields. Some of the terms include mixings arising from terms like \( \text{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] \):

\[ \pi_N - a_{1N}^\mu : -g_1 \phi_N a_{1N}^\mu \partial_\mu \pi_N, \]
\[ \pi - a_1^\mu : -g_1 \phi_N (a_1^\mu + \partial_\mu \pi^- + a_1^{\mu 0} \partial_\mu \pi^0) + \text{h.c.}, \]
\[ \pi_S - a_{1S}^\mu : -\sqrt{2} g_1 \phi_S a_{1S}^\mu \partial_\mu \pi_S, \]
\[ K_S - K^{\ast\mu}_\mu : \frac{ig_1}{2} (\sqrt{2} \phi_S - \phi_N) (K^{\ast 0}_\mu \partial_\mu K^0_S + K^{\ast -}_\mu \partial_\mu K^+_S) + \text{h.c.}, \]
\[ K - K^\mu_1 : -\frac{g_1}{2} (\phi_N + \sqrt{2} \phi_S) (K^\mu_1 \partial_\mu \bar{K}^0 + K_1^{\mu +} \partial_\mu K^-) + \text{h.c.} \]

Diagonalization \rightarrow \text{Wave function renormalization}
Determination of the parameters of the Lagrangian

16 unknown parameters \((m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F, g_V, g_A)\) \(\rightarrow\) Determined by the min. of \(\chi^2:\)

\[
\chi^2(x_1, \ldots, x_N) = \sum_{i=1}^{M} \left[ \frac{Q_i(x_1, \ldots, x_N) - Q_{i}^{\text{exp}}}{\delta Q_i} \right]^2 ,
\]

where \((x_1, \ldots, x_N) = (m_0, \lambda_1, \lambda_2, \ldots), Q_i(x_1, \ldots, x_N)\) calculated from the model, while \(Q_{i}^{\text{exp}}\) taken from the PDG

multiparametric minimalization \(\rightarrow\) MINUIT

- PCAC \(\rightarrow\) 2 physical quantities: \(f_\pi, f_K\)
- Tree-level masses \(\rightarrow\) 15 physical quantities:
  \[m_u/d, m_s, m_\pi, m_\eta, m_\eta', m_K, m_\rho, m_\Phi, m_{K^*}, m_{a_1}, m_{f_1^H}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}\]
- Decay widths \(\rightarrow\) 12 physical quantities:
  \[
  \Gamma_{\rho \rightarrow \pi \pi}, \Gamma_{\Phi \rightarrow KK}, \Gamma_{K^* \rightarrow K \pi}, \Gamma_{a_1 \rightarrow \pi \gamma}, \Gamma_{a_1 \rightarrow \rho \pi}, \Gamma_{f_1 \rightarrow KK^*}, \Gamma_{a_0}, \Gamma_{K_S \rightarrow K \pi},
  \Gamma_{f_0^L \rightarrow \pi \pi}, \Gamma_{f_0^L \rightarrow KK}, \Gamma_{f_0^H \rightarrow \pi \pi}, \Gamma_{f_0^H \rightarrow KK},
  \]
- \(T_c = 155\) MeV from lattice
### Results

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<th>Parameter</th>
<th>Cal (GeV)</th>
<th>Mass</th>
<th>Cal (GeV)</th>
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### Parameters

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<td>$\phi_S$ [GeV]</td>
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- with this set
  $f_0^l = 0.2837$ GeV
Polyakov loops in Polyakov gauge

Polyakov loop variables: \( \Phi(\vec{x}) = \frac{\text{Tr}_c L(\vec{x})}{N_c} \) and \( \bar{\Phi}(\vec{x}) = \frac{\text{Tr}_c \bar{L}(\vec{x})}{N_c} \) with \( L(x) = \mathcal{P} \exp \left[ i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right] \)

→ signals center symmetry (\( \mathbb{Z}_3 \)) breaking at the deconfinement transition

low \( T \): confined phase, \( \langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle = 0 \)

high \( T \): deconfined phase, \( \langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle \neq 0 \)

Polyakov gauge: the temporal component of the gauge field is time independent and can be gauge rotated to a diagonal form in the color space

\[ G_{4,d}(\vec{x}) = \phi_3(\vec{x}) \lambda_3 + \phi_8(\vec{x}) \lambda_8; \quad \lambda_3, \lambda_8 : \text{Gell-Mann matrices.} \]

In this gauge the Polyakov loop operator is

\[ L(\vec{x}) = \text{diag}(e^{i\beta \phi_+(\vec{x})}, e^{i\beta \phi_-(\vec{x})}, e^{-i\beta (\phi_+(\vec{x}) + \phi_-(\vec{x}))}) \]

where \( \phi_\pm(\vec{x}) = \pm \phi_3(\vec{x}) + \phi_8(\vec{x})/\sqrt{3} \)
Polyakov loop

Improved Polyakov loops potential


\[
\mathcal{U}_{\log}(\Phi, \bar{\Phi}) = -\frac{1}{2} a(T)\Phi\bar{\Phi} + b(T) \ln \left[ 1 - 6\Phi\bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi\bar{\Phi})^2 \right]
\]

with

\[
a(T) = a_0 + a_1 \frac{T_0}{T} + a_2 \frac{T_0^2}{T^2}, \quad b(T) = b_3 \frac{T_0^3}{T^3}
\]

\(\mathcal{U}(\Phi, \bar{\Phi})\) models the free energy of a pure gauge theory

Within FRG, the glue potential \(U_{\text{glue}}(\Phi, \bar{\Phi})\) coming from the gauge dof propagating in the presence of dynamical quarks can be matched to the potential \(U_{\text{YM}}(\Phi, \bar{\Phi})\) of the SU(3) YM theory by relating the reduced temperatures:

\[
\frac{U_{\text{glue}}}{T^4}(\Phi, \bar{\Phi}, t_{\text{glue}}) = \frac{U_{\text{YM}}}{(T_{\text{YM}})^4}(\Phi, \bar{\Phi}, t_{\text{YM}}(t_{\text{glue}})), \quad t_{\text{YM}}(t_{\text{glue}}) \approx 0.57 t_{\text{glue}}
\]

\[
t_{\text{glue}} \equiv \frac{T - T_c^{\text{glue}}}{T_c^{\text{glue}}}, \quad t_{\text{YM}} \equiv \frac{T_{\text{YM}} - T_0^{\text{YM}}}{T_0^{\text{YM}}}, \quad T_c^{\text{glue}} \in (180, 270)\text{MeV}
\]

L.M.Haas et al., PRD 87, 076004 (2013)
**Polyakov loop**

**Polyakov loop potential**

"Color confinement"

\[ \langle \Phi \rangle = 0 \rightarrow \text{no breaking of } \mathbb{Z}_3 \]

one minimum

"Color deconfinement"

\[ \langle \Phi \rangle \neq 0 \rightarrow \text{spontaneous breaking of } \mathbb{Z}_3 \]

minima at \(0, 2\pi/3, -2\pi/3\)

one of them spontaneously selected

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from H. Hansen et al., PRD75, 065004 (2007)
**Effects of Polyakov loops on FD statistics**

Inclusion of the Polyakov loop modifies the Fermi-Dirac distribution function

\[
\begin{align*}
    f(E_p - \mu_q) & \rightarrow f_\Phi^+(E_p) = \frac{\left(\Phi + 2\Phi e^{-\beta(E_p-\mu_q)}\right) e^{-\beta(E_p-\mu_q)} + e^{-3\beta(E_p-\mu_q)}}{1 + 3 \left(\Phi + 2\Phi e^{-\beta(E_p-\mu_q)}\right) e^{-\beta(E_p-\mu_q)} + e^{-3\beta(E_p-\mu_q)}} \\
    f(E_p + \mu_q) & \rightarrow f_\Phi^-(E_p) = \frac{\left(\Phi + 2\Phi e^{-\beta(E_p+\mu_q)}\right) e^{-\beta(E_p+\mu_q)} + e^{-3\beta(E_p+\mu_q)}}{1 + 3 \left(\Phi + 2\Phi e^{-\beta(E_p+\mu_q)}\right) e^{-\beta(E_p+\mu_q)} + e^{-3\beta(E_p+\mu_q)}}
\end{align*}
\]

\(\Phi, \bar{\Phi} \rightarrow 0 \Rightarrow f_\Phi^\pm(E_p) \rightarrow f(3(E_p \pm \mu_q)) \quad \Phi, \bar{\Phi} \rightarrow 1 \Rightarrow f_\Phi^\pm(E_p) \rightarrow f(E_p \pm \mu_q)\)

three-particle state appears: mimics confinement of quarks within baryons

at \(T = 0\) there is no difference between models with and without Polyakov loop
$T/\mu_B$ dependence of the condensates

$\Omega$: grand canonical potential

\[
\frac{\partial \Omega}{\partial \phi} = \frac{\partial \Omega}{\partial \bar{\phi}} \bigg|_{\phi_N=\phi_N, \phi_S=\phi_S} = 0
\]

\[
\frac{\partial \Omega}{\partial \phi_N} = \frac{\partial \Omega}{\partial \phi_S} \bigg|_{\phi, \bar{\phi}} = 0, \quad \text{(after the SSB)}
\]

**Hybrid approach:** fermions at one-loop, mesons at tree-level (their effects are much smaller)
Extremum equations for $\phi_{N/S}$ and $\Phi$, $\bar{\Phi}$

**Masses**

$$M_{i,ab}^2 = \left. \frac{\partial^2 \Omega(T, \mu_f)}{\partial \varphi_i,a \partial \varphi_i,b} \right|_{\text{min}} = m_{i,ab}^2 + \Delta_0 m_{i,ab}^2 + \Delta_T m_{i,ab}^2,$$

$m_{i,ab}^2 \longrightarrow$ tree-level mass matrix,

$\Delta_0/T m_{i,ab}^2 \longrightarrow$ fermion vacuum/thermal fluctuation,

$$\Delta_0 m_{i,ab}^2 = \left. \frac{\partial^2 \Omega_{\text{vac}}^{qq}}{\partial \varphi_i,a \partial \varphi_i,b} \right|_{\text{min}} = - \frac{3}{8\pi^2} \sum_{f=u,d,s} \left[ \left( \frac{3}{2} + \log \frac{m_f^2}{M^2} \right) m_{f,a}^2 m_{f,b}^2 + m_f^2 \left( \frac{1}{2} + \log \frac{m_f^2}{M^2} \right) m_{f,ab}^2 \right],$$

$$\Delta_T m_{i,ab}^2 = \left. \frac{\partial^2 \Omega_{\text{th}}^{qq}}{\partial \varphi_i,a \partial \varphi_i,b} \right|_{\text{min}} = \frac{1}{6} \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_f(p)} \left[ (f_f^+(p) + f_f^-(p)) \left( m_{f,a}^2 m_{f,b}^2 - \frac{m_{f,a}^2 m_{f,b}^2}{2E_f^2(p)} \right) + (B_f^+(p) + B_f^-(p)) \frac{m_{f,a}^2 m_{f,b}^2}{2TE_f(p)} \right],$$

where $m_{f,a}^2 \equiv \partial m_f^2 / \partial \varphi_i,a$, $m_{f,ab}^2 \equiv \partial^2 m_f^2 / \partial \varphi_i,a \partial \varphi_i,b$
Features of our approach

- D.O.F’s: scalar, pseudoscalar, vector, axial vector nonets,
- Polyakov loop variables, $\Phi, \Phi$ with $U_{YM}$ or $U_{glue}$
- $u,d,s$ constituent quarks, $(m_u = m_d)$
- no mesonic fluctuations included in the grand canonical potential:
  $$\Omega(T, \mu_q) = -\frac{1}{\beta V} \ln(Z)$$
- Fermion vacuum and thermal fluctuations
- quarks do not couple to (axial) vector meson yet
- Four order parameters $(\phi_N, \phi_S, \Phi, \Phi) \rightarrow$ four $T/\mu$-dependent equations
- thermal contribution of $\pi, K, f_0^L$ included in the pressure
$\phi_{N/S}(T), \Phi/\bar{\Phi}(T)$ with Polyakov loop $m_{f_0}^L = 1326$ MeV

Condensates and Polyakov loop variables with vacuum fluctuations
With low mass scalars, \( m_{f_0^L} = 300 \text{ MeV} \)

chiral symmetry is restored at high \( T \) as the chiral partners \((\pi, f_0^L), (\eta, a_0)\) and \((K, K_0^*), (\eta', f_0^H)\) become degenerate

\( U(1)_A \) symmetry is not restored, as the axial partners \((\pi, a_0)\) and \((\eta, f_0^L)\) do not become degenerate
Mass pattern in the $\eta$, $\eta'$ sector

Our pattern: $m_\eta \leq m_{\eta_N} < m_{\eta_S} \leq m_{\eta'}$ in contrast to others

Schaefer, PRD79 014018, Tiwari PRD88, 074017
\[ \Delta = \frac{(\Phi_N - \frac{h_N}{h_S} \Phi_S)_T}{(\Phi_N - \frac{h_N}{h_S} \Phi_S)_{T=0}} \]

Subtracted chiral condensate

\[ U_{\text{glue}} \text{ with } T_{\text{glue}}^{\text{glue}} \in (210 - 240) \text{MeV} \]
gives good agreement with the lattice result of Borsanyi et al., JHEP 1009, 073 (2010)

- Lattice shows smooth transition
- Our result is completely off
- Renormalization of the Polyakov loop may explain part of the discrepancy
  Andersen et al., PRD92, 114504
We include mesonic thermal contribution to $p$ for $(\pi, K, f_0^I)$

$$\Delta p(T) = -nT \int \frac{d^3q}{(2\pi)^3} \ln(1 - e^{-\beta E(q)}), \quad E(q) = \sqrt{q^2 + m^2}$$

- pressure: $p(T, \mu_q) = \Omega_H(T = 0, \mu_q) - \Omega_H(T, \mu_q)$
- entropy density: $s = \frac{\partial p}{\partial T}$
- quark number density: $\rho_q = \frac{\partial p}{\partial \mu_q}$
- energy density: $\epsilon = -p + Ts + \mu_q \rho_q$
- scaled interaction measure: $\frac{\Delta}{T^4} = \frac{\epsilon - 3p}{T^4}$
- speed of sound at $\mu_q = 0$: $c_s^2 = \frac{\partial p}{\partial \epsilon}$
Normalized pressure

we use $U_{glue}$ with $T_{c}^{glue} = 270$ MeV

pion dominates at low $T$

at high $T$ pressure overshoots the lattice data

overshooting increases with decreasing $T_{c}^{glue}$

lattice: Borsányi et al., JHEP 1011, 077 (2010)
Observables

interaction measure

speed of sound

$T$ dependence of the order parameters

$U_{YM}, T_0=182$ MeV, only $\pi$

$U_{YM}, T_0=182$ MeV

$U_{glue}, T_c^{glue}=150$ MeV

$U_{glue}, T_c^{glue}=182$ MeV

$U_{glue}, T_c^{glue}=210$ MeV

$U_{glue}, T_c^{glue}=240$ MeV

$U_{glue}, T_c^{glue}=270$ MeV

lattice

SB: $1/3$

$c_s^2(t), p/\epsilon$

$U_{YM}, T_0=182$ MeV

$U_{glue}, T_c^{glue}=182$ MeV

$U_{glue}, T_c^{glue}=210$ MeV

$U_{glue}, T_c^{glue}=240$ MeV

$U_{glue}, T_c^{glue}=270$ MeV, only $\pi$

lattice
\( T - \mu_B \) phase diagram

- We use \( U^{\text{glue}} \) with \( T_c^{\text{glue}} = 210 \) MeV
- Freeze-out curve from Cleymans et al., J.Phys.G32, S165
- Curvature at \( \mu_B = 0 \) \( \kappa = 0.0193 \), close to the lattice value \( \kappa = 0.020(4) \)

(Cea et al., PRD93, 014507)
$T - \rho_B$ phase diagram

black: params of PRD93, 114014
Isentropic trajectories in the $T - \mu_B$ plane

our model, where $\mu^\text{CEP}_B > 850\text{MeV}$

same qualitative behavior of the isentropic trajectories for $\mu_B \leq 400\text{ MeV}$

$\Rightarrow$ indication that in the lattice result there is no CEP in this region of $\mu_B$

lattice (analytic continuation)

Günther et al., arXiv:1607.02493
Summary and Conclusions

- The thermodynamics of the ePQM was studied after parametrizing of the model with a modification of the method used in Parganlija et al., PRD87, 014011
- 40 possible assignments of the scalars to the nonet states were investigated. Lowest $\chi^2$ for $a_0^q \rightarrow a_0(980), K_0^*, qq \rightarrow K_0^*(980), f_0^l, qq \rightarrow f_0(500), f_0^h, qq \rightarrow f_0(980)$
- The phase transition temperature requires low mass ($\leq 400$ MeV) $f_0$
- For the best set of parameters CEP was found in the $T - \mu_B$ plane
- The $T$-dependence of various thermodynamical observables measured on the lattice is reasonable well reproduced with an improved Polyakov loop potential. L.M. Hass et al., PRD87, 076004
- The model’s predictions are unrealistic at large $\mu_B$
→ **To do . . .**

→ Improve the vacuum phenomenology by tetraquarks (and glueballs)
→ coupling the quarks to the (axial)vectors
→ including mesonic fluctuations
→ find a way to improve the high density behaviour
Thank you for your attention!